Data Structures - Assignment no. 1

Date due: 23/7/01

Remarks:

- If you fail to understand a question, or find a question erroneous, please contact me at `puzne@tau.ac.il`.
- Unless you are explicitly asked otherwise (as in question no. 1 herein), please describe algorithms only to the extent which makes them clear to a reasonable reader. You may use plain language, or a pseudo-code, whichever is more convenient.

1. Give a detailed algorithm that receives an array A as input, and outputs its maximal element. Your algorithm should consist only of operations that are applicable in one step of the RAM machine. Explain how and where the input is put, and where the output is written.

2. Consider the following computational task. The input is a pointer to the beginning of a singly linked list, where the 'next' pointer of the final node in the list is not null – instead it points to one of the other nodes in the list, creating a loop. The goal is to return a pointer to the final node in the list. Give an algorithm which accomplishes this task, without making any changes to the given list, at any time. If you can, give a linear time algorithm (namely the number of operations should be proportional to the length of the list). **Hint:** First find an algorithm that just finds one element in the list that is within the loop.

3. Consider the following abstract data type, called **abstract array**, and supports the following operations:
   - **init(n)** - initializes the structure to contain n elements.
   - **set(i, x)** - sets the value of the ith element to x (i should be between 1 and n).
   - **get(i)** - returns the value of the ith element.

   (a) Show an implementation of an abstract array, that requires a constant time for each operation.

   (b) A **resetable array** is an abstract array that also supports the operation **setall(x)**, which sets the values of *all* the elements to x. Show an implementation of an resetable array that performs all operations other than **init** in constant time.

   (c) (*) Implement a resetable array so that all operations, including initiation, take constant time.

4. Prove or disprove the following statement. Let $f : \mathbb{N} \to \mathbb{N}$ and $g : \mathbb{N} \to \mathbb{N}$ be strictly increasing functions. Then either $f \prec g$ or $g \prec f$.

5. Rank the following functions according to their order of growth. If any of the functions are of the same order, specify that in your answer.
   n, n^2, $\frac{n^4+1}{n^2+1}$, $\log_2 n$, $[\log_2(\log_2 n)]^{\log_2(\log_2 n)}$, $\log_2^2 n$, $n^{2\log_2 n}$, 2^n, 3^n, $(\log_2 n)^{\log_2 n}$, $2^{\log_2^2 n}$, $n^{\log_2 \log_2 n}$, $n!$, $(n+1)!$, n^n, $2^{\log_3 n}$, $n^{\frac{2n+1}{n+1}}$, $(\log_2 n)^{\frac{\log_2 n}{\log_2(\log_2 n)}}$.