Order Optimal Information Spreading Using Algebraic Gossip

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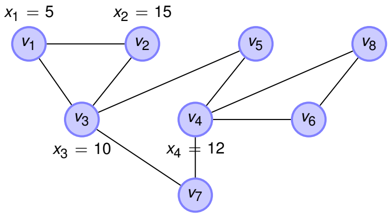
Motivation

- Wireless (sensor) networks and peer-to-peer networks need efficient algorithms for information dissemination.
- In such networks, there is no central management entity, thus local, distributed algorithms are needed.
- Network Coding with gossip algorithms (a.k.a. Algebraic Gossip) will help us to achieve faster information dissemination.
- We look at: k nodes want to disseminate their value to all other nodes in the network.

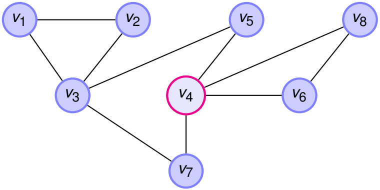
Information Spreading - The k-Dissemination Problem



- A network represented by a graph $G(V, E)$. $V = \{v_1, v_2, \ldots, v_n\}$
- $k \leq n$ values $\{x_1, x_2, \ldots, x_k\}$ need to be distributed to all nodes
- A node knows only its neighbors
- Limited messages size

Gossip Algorithm – The Way Information is Spread

In each round **every node** takes a **gossip action**



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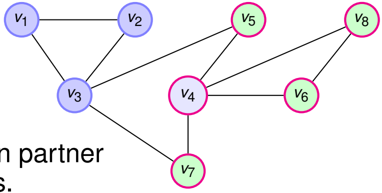

- Gossip Algorithm:
  1. Determines a communication partner (randomly) among neighbors.
     - Uniform gossip.
     - Non uniform gossip.

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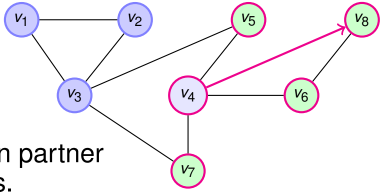
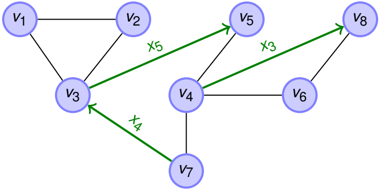


- Gossip Algorithm:
  1. Determines a communication partner (randomly) among neighbors.
     - Uniform gossip.
     - Non uniform gossip.
  2. Determines how the message is sent.
     - **PUSH** – a message is sent to the partner.
     - **PULL** – a message is sent from the partner.
     - **EXCHANGE** - PUSH and PULL.

Algebraic Gossip is Based on Random Linear Network Coding

instead of sending randomly chosen values...



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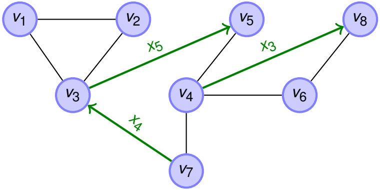

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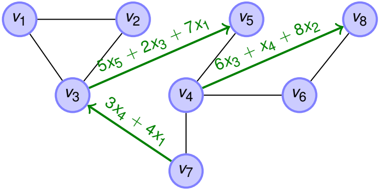

every message – linear equation: msg $= \sum a_i x_i$

- Nodes (routers) can manipulate packets.
- All operations are in a field $\mathcal{F}$ so messages size is (almost) the same in both cases.

Algebraic Gossip is Based on Random Linear Network Coding

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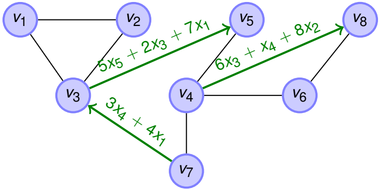

every message – linear equation

linear equations are stored in a matrix form:

$$\begin{bmatrix} 4 & 3 & 7 & 6 \\ 2 & 0 & 0 & 7 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 22 \\ 45 \\ 78 \\ 30 \end{bmatrix}$$

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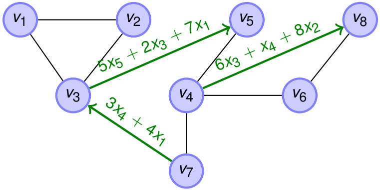

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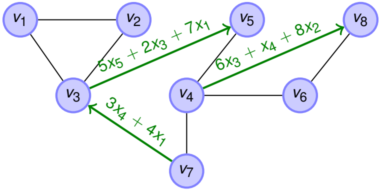

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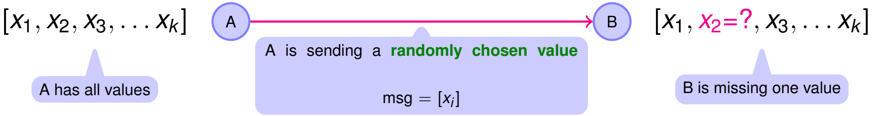
once a node has rank k – it finishes

only **helpful** messages are stored

messages that increase the rank of the matrix

So, Why Algebraic Gossip is Faster?

Without Algebraic Gossip (Random Message Selection)



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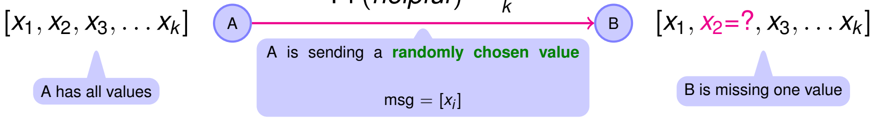
Without Algebraic Gossip (Random Message Selection)

$$\Pr(helpful) = \frac{1}{k}$$



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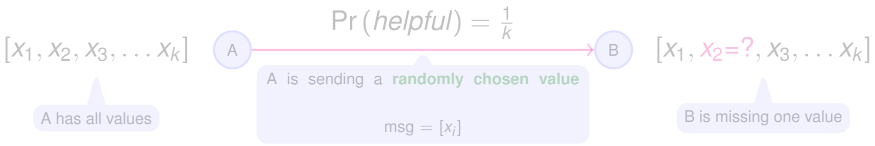

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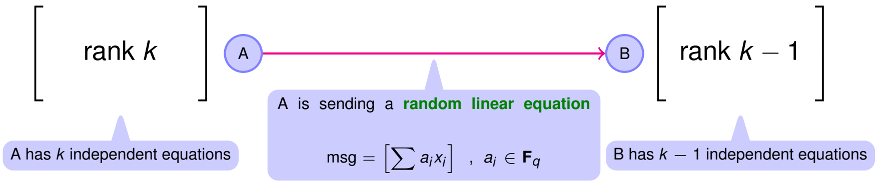

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$[x_1, x_2, x_3, \dots x_k]$ A → B $[x_1, x_2{=}?, x_3, \dots x_k]$

A has all values

A is sending a **randomly chosen value**

msg $= [x_i]$

B is missing one value

With Algebraic Gossip (Random Linear Equations)

$$\Pr(helpful) = 1 - \frac{1}{q}$$

$\begin{bmatrix} \text{rank } k \end{bmatrix}$ A → B $\begin{bmatrix} \text{rank } k-1 \end{bmatrix}$

A has k independent equations

A is sending a **random linear equation**

msg $= \left[\sum a_i x_i\right]$, $a_i \in \mathbf{F}_q$

B has $k-1$ independent equations

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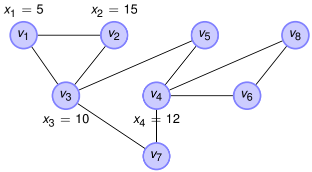
$$\Pr(\textit{helpful}) = \frac{q^k - q^{k-1}}{q^k} = 1 - \frac{1}{q}$$

Research Goal – Optimal Protocol for k-Dissemination Problem



1. Analyze uniform algebraic gossip for k-dissemination
   - Is it optimal?
   - For which graphs?
2. Study non-uniform gossip to achieve optimal k-dissemination

Related Work on Algebraic Gossip

- Trivial lower bound – $\Omega(k)$, kn messages needed to be delivered so k rounds.

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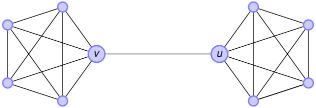
- [BAL. ISIT10] – n-dissemination. Upper bound for any graph: $O(\Delta n)$. Tight bound of $\Theta(n)$ for *constant degree graphs*. Worst case graph for algebraic gossip (barbell): $\Omega(n^2)$.



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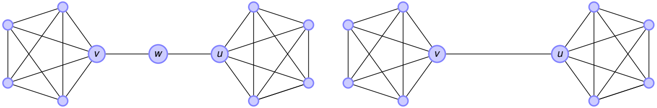
- Open question: What graph property capture the stopping time?



Related Work on Algebraic Gossip

- [Haeupler. STOC11] – k-dissemination. Conductance and expansion based arguments. Two parameters: γ and λ.
  - Tight bound for the case $k = \Omega(n)$: $\Theta(n/\gamma)$
  - For $k < o(n)$: $O(k/\gamma + \log^2 n/\lambda)$. The bound is not tight for e.g., line: $O(k + n \log^2 n)$, grid: $O(k + \sqrt{n} \log^2 n)$, binary tree: $O(k + n \log^2 n)$
  - Gave also results for dynamic networks

1st Result: k-Dissemination With Uniform Algebraic Gossip

Theorem 1

For any graph with n nodes, diameter D, and maximum degree Δ, stopping time of **uniform** algebraic gossip is $O(\Delta(k + \log n + D))$ with high probability.

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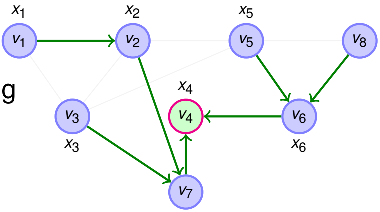
- Tight for e.g., Line, Cycle, Grids, Binary Trees, etc.
- The result holds for any gossip variation: Push, Pull, Exchange.
- When does the uniform algebraic gossip perform bad? e.g., $\Omega(kn)$ for a barbell graph.

2nd Result: k-Dissemination With TAG

- Construct a spanning tree of the graph using some gossip spanning tree protocol $\mathcal{S}$.



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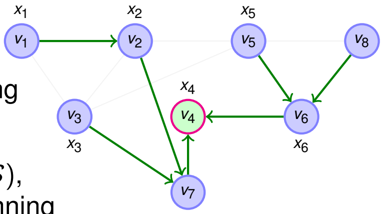

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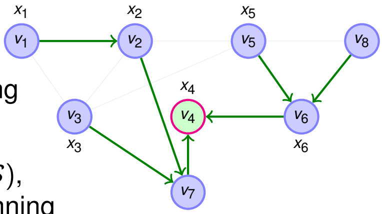

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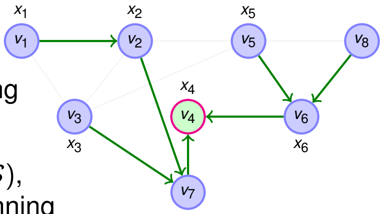

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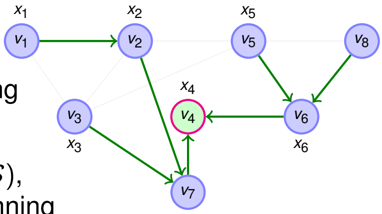

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  - Spanning tree can be constructed e.g., by a randomize broadcast protocol, or more sophisticated method.
- Once the tree is constructed, every node knows its parent.
- Perform algebraic gossip, where every node uses a single communication partner – its parent.
  Notice, we have here **non-uniform** algebraic gossip.

2nd Result: k-Dissemination With TAG

Theorem 2

For any graph with n nodes,
stopping time of TAG protocol is $O(k + \log n + d(\mathcal{S}) + t(\mathcal{S}))$
with high probability, where:
$t(\mathcal{S})$ – stopping time of the gossip spanning tree protocol $\mathcal{S}$.
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Corollary 2

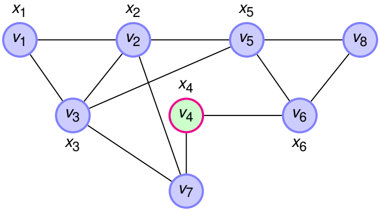
For any graph with n nodes, and for $k = \Omega(k)$,
TAG protocol is **order optimal** for k-dissemination task, i.e., the stopping time is $\Theta(n)$ with high probability.

Proof Overview

k-dissemination with **uniform** algebraic gossip

Proof Overview – Converting a Graph to a System of Queues

initial graph with an arbitrary node v_4



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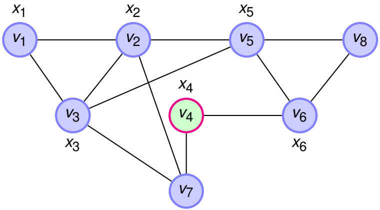

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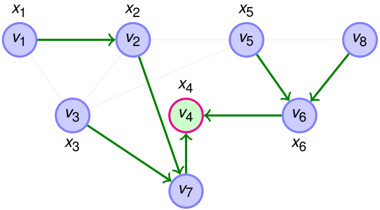
BFS spanning tree rooted at v_4



we **ignore** the messages coming from other edges

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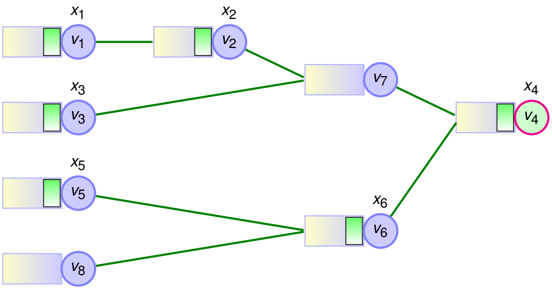

customers are **helpful** messages

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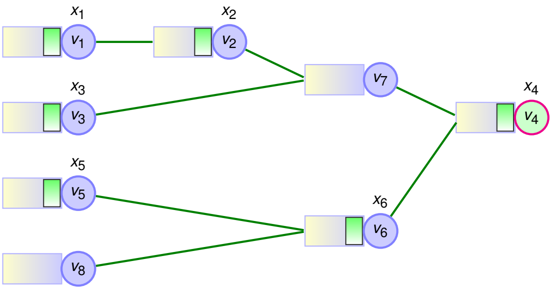

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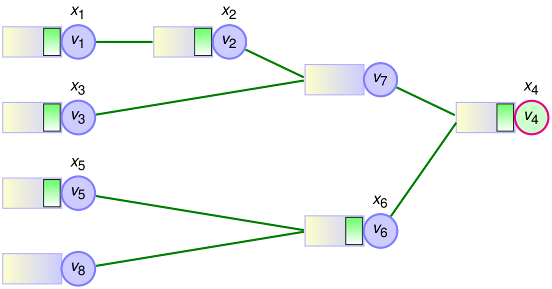

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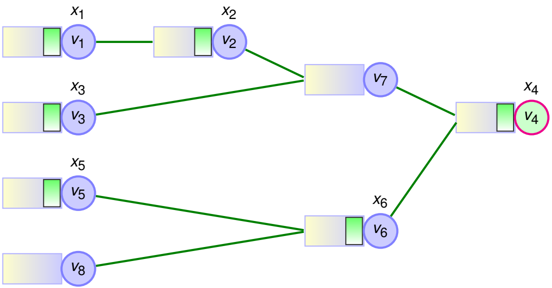

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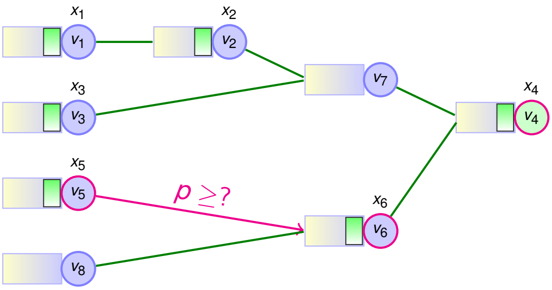

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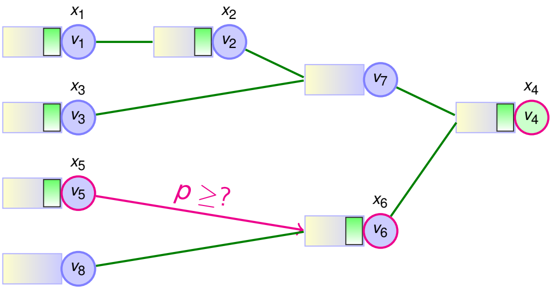

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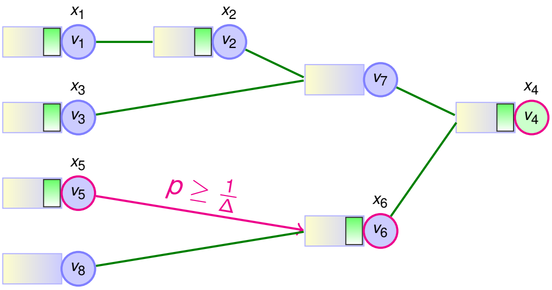

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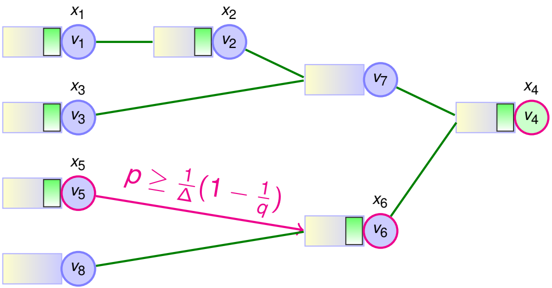

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the message will be **helpful** w.p. $\geq (1 - \frac{1}{q})$

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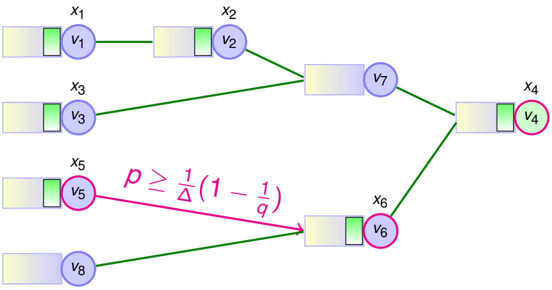


customers are **helpful** messages

initially, some nodes have **helpful** messages

customer arriving at some node, increases its rank by 1

once v_4 receives k **helpful** messages it finishes

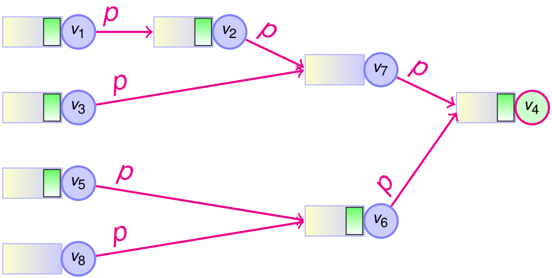
in a given **round**, v_5 wakes up exactly once

v_5 chooses v_6 as a partner w.p. $\geq \frac{1}{\Delta}$

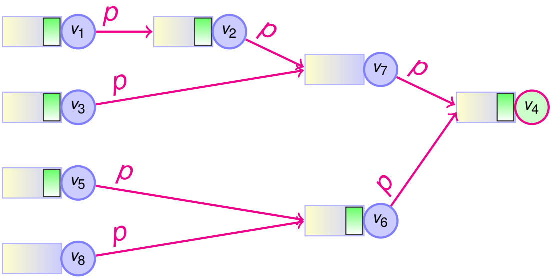
the message will be **helpful** w.p. $\geq (1 - \frac{1}{q})$

service time is geometrically distributed with p

Proof Overview – Exponential Servers Instead of Geometric



Proof Overview – Exponential Servers Instead of Geometric



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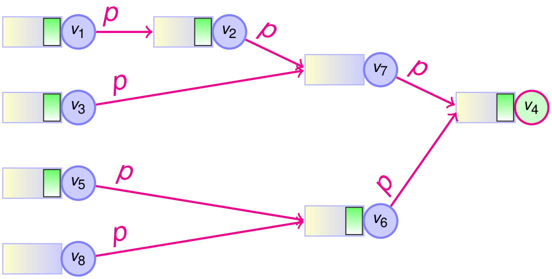

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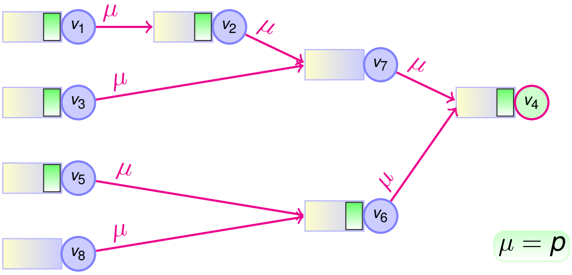

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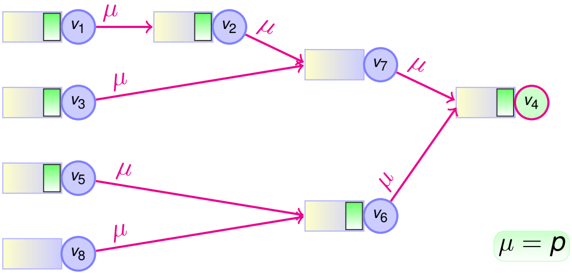
we replace servers, thus increasing the stopping time

Line is Slower Than Tree

When does the last customer leave the system?



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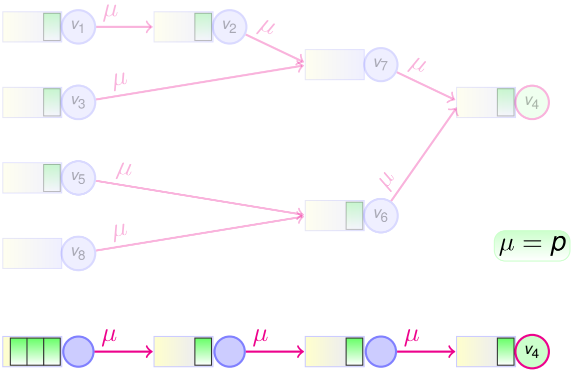
Reduce tree to line

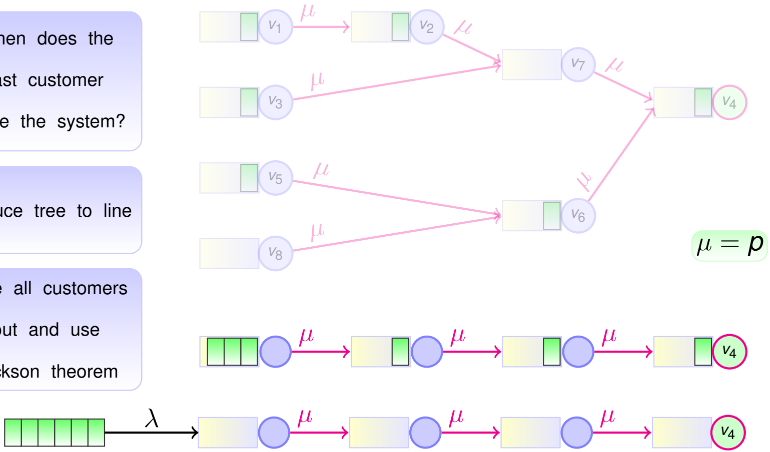


Line is Slower Than Tree

When does the last customer leave the system?

Reduce tree to line

Take all customers out and use Jackson theorem



Line of D Queues

- **Jackson's Theorem:** If utilization at every queue is less than 1, the equilibrium state distribution of number of customers in each queue exists and it is given by:

For state $(k_1, k_2, \ldots, k_n)$, and utilization $\rho_i = \frac{\lambda_i}{\mu_i}$: $\pi(k_1, k_2, \ldots, k_n) = \prod_{i=1}^{n} \rho_i^{k_i}(1 - \rho_i)$.

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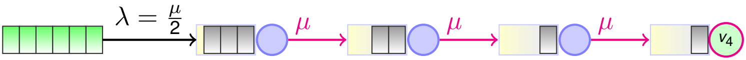

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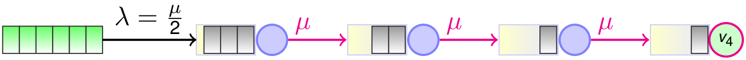

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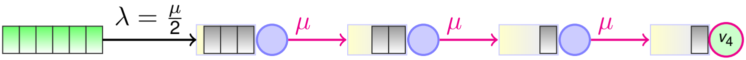

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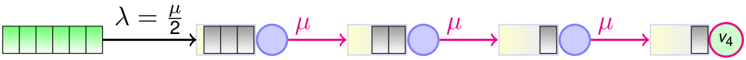
- Time to cross one MM1 queue in the stationary state is exponentially distributed with $\mu - \lambda = \mu/2$.
- So, the time needed to cross D MM1 queues is $O((d + \log n)/\mu)$, where $\log n$ is needed for high probability.



Last customer leaves after: $O((k + \log n + D)/\mu) = O(\Delta(k + \log n + D))$ rounds

Proof Overview

k-dissemination with TAG
Tree-Based Algebraic Gossip

TAG – Tree Based Algebraic Gossip Protocol

- The proof is also based on analyzing a tree network of queues
- The uniform gossip bound is

$$O(\Delta(k + \log n + D))$$

- The TAG based bound is:

$$O(t(\mathcal{S}) + k + \log n + d(\mathcal{S}))$$

- In the synchronous time model $t(\mathcal{B}) \geq d(\mathcal{B})$. ($\mathcal{B}$ is a broadcast that builds a tree)
- For *Round Robin* Broadcast, $t(\mathcal{B}_{\mathcal{RR}}) = O(n)$ so for $k = \Omega(n)$ the stopping time of TAG is $\Theta(n)$

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- Thank you!

Deb, S., Médard, M., and Choute, C. (2006).
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