Experimental Approaches in Computer Science

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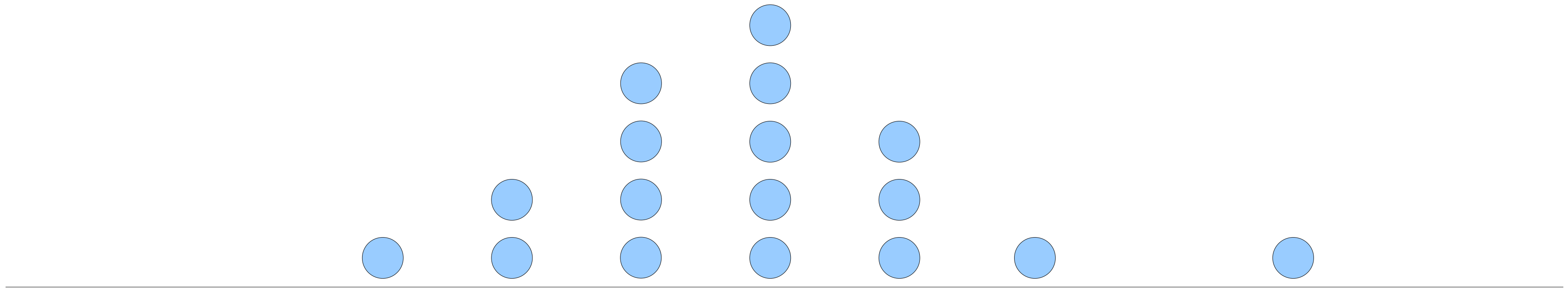
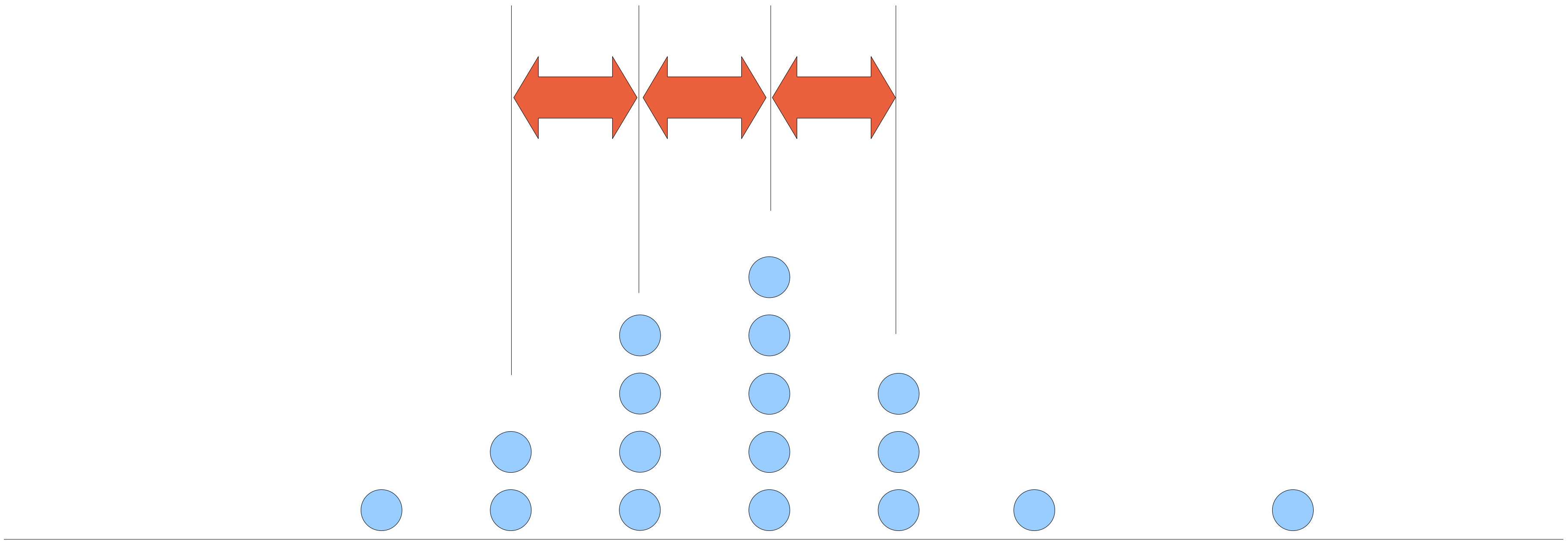
Lecture 3 – Measurements

Assume we start taking measurements of something:



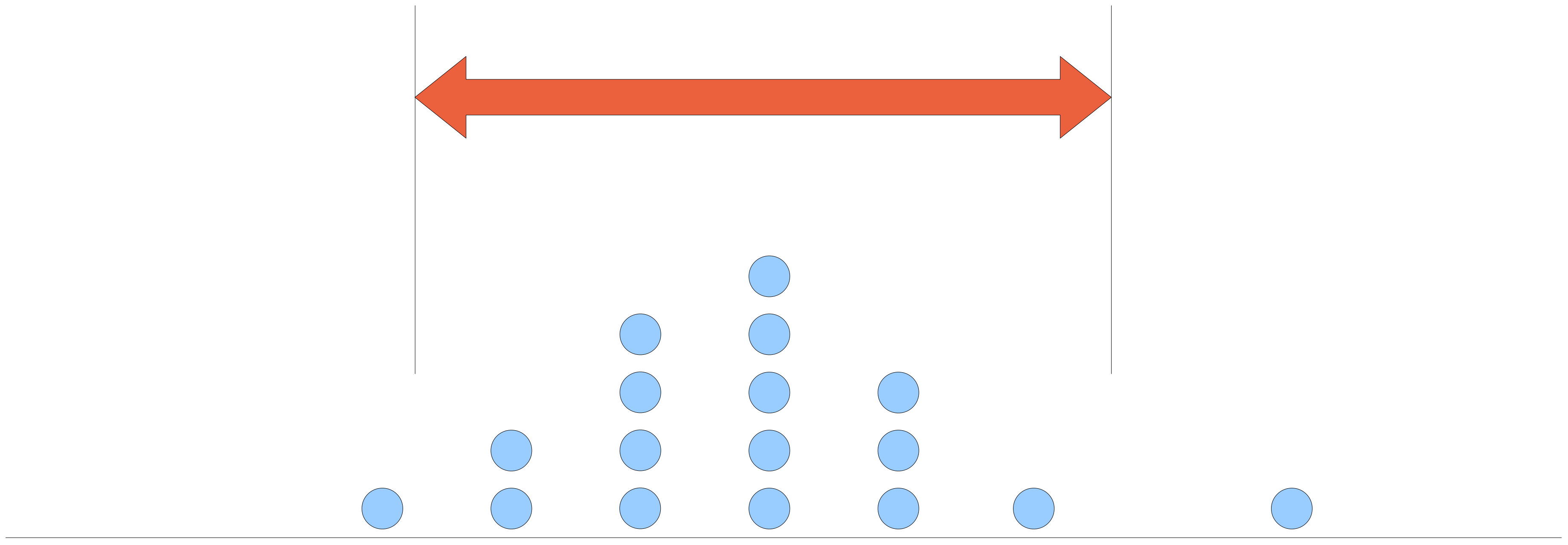
Such measurements are limited by the resolution of our measurement apparatus: the smallest difference between measurements

Example: measuring time in milliseconds



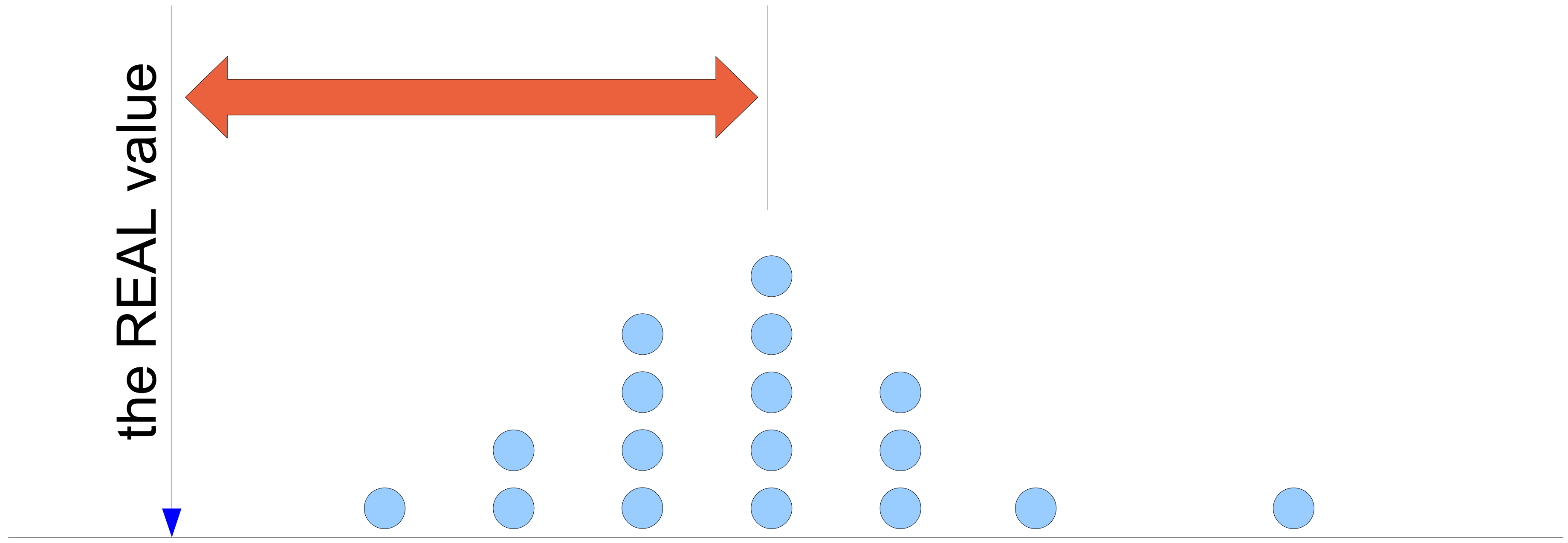
Another problem is the uncertainty in the measurements: if we repeat them, we get somewhat different results

This is caused by random errors that reflects the (im)precision of the measurement



A third issue is the accuracy of the measurement: how far it is from the “real” value

This reflects the systematic error in the measurement



Systematic errors are systematic – they always have the same effect

Example: when measuring the time to perform an action, the overhead of the measurement itself is added to the result

Part of the design should be to identify systematic errors and factor them out

A special problem is warmup or hysteresis-type errors, where the outcome depends on history; for example, the first measurement could be different from subsequent ones

Random errors are random – they may have different effects

Example: when measuring the time to perform an action, the result may depend on cache state, interrupts from the network, and competition from other processes

This can be analyzed statistically

In extreme cases, interference leads to outliers that should be ignored

Model of random errors:

Assume the error is a combination of multiple effects, each contributing ±ε

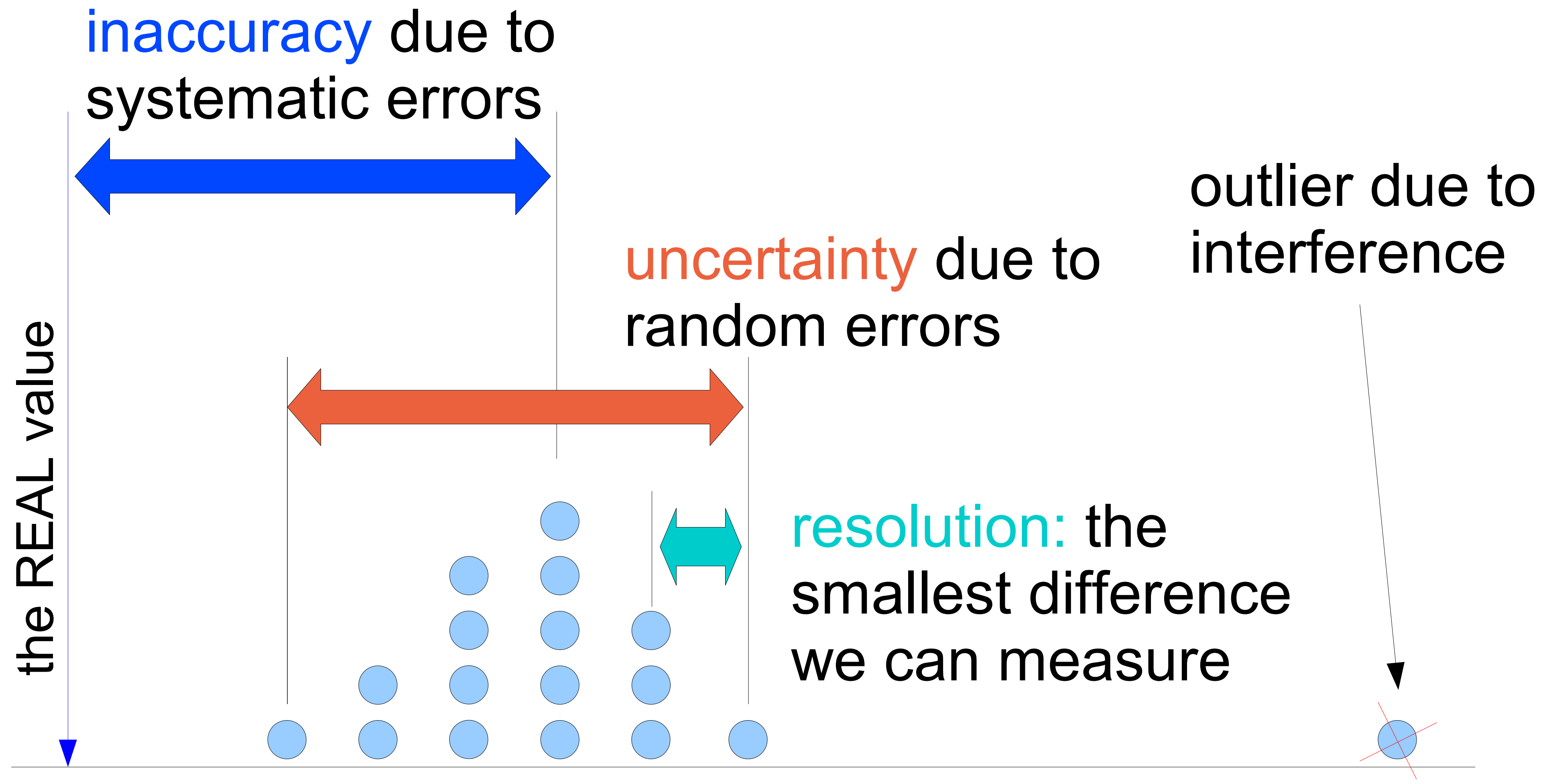
If we have *n* such effects, the outcome will have a binomial distribution

$$p(X + k\epsilon) = \binom{n}{\frac{n+k}{2}} \frac{1}{2^n}$$

For large *n*, this can be approximated by a normal distribution

Calculation of confidence intervals is based on assumption of a normal distribution

Summary:


inaccuracy due to systematic errors
outlier due to interference
uncertainty due to random errors
the REAL value
resolution: the smallest difference we can measure

Measuring Time

A computer (specifically, Intel PC) has three time sources:

Timer interrupts

Time stamp counter

Time server and NTP protocol

Timer interrupts:

Cause a CPU interrupt, and running the clock interrupt handler

Come at a certain frequency (e.g. 250 Hz) from an external timer

The system counts how many interrupts (called ticks or jiffies) have occurred

Reasonably accurate measure of wallclock time

Typical resolution of several milliseconds (maximal resolution is microseconds)

Time stamp counter:

A special register that counts the cycles since the machine was booted

Can be read by a special assembler instruction

Rate depends on the CPU clock rate

Can drift e.g. when temperature rises

Can change due to power considerations, especially on laptops (reduce speed to save energy)

Nanosecond (cycle) resolution, but need many cycles to take a reading

NTP (Network Time Protocol)

Most networks have a designated NTP server

The NTP server gets time from some standard source

All nodes in the network synchronize with the NTP server

Effect is to update the system time

Can lead to a jump in time

Jump can also be backwards

Alternative is to change the time gradually

Linux gettimeofday()

Upon each clock interrupt

Update notion of real time as per the external source

Synchronize with the time stamp counter

When called, read the current time stamp counter and extrapolate from the previous clock interrupt

Combines different timing sources

Report result in microsecond resolution (API)

Making a Measurement

The framework:

Measurement of some computer activity or operation

Done from user level

With no specialized tools

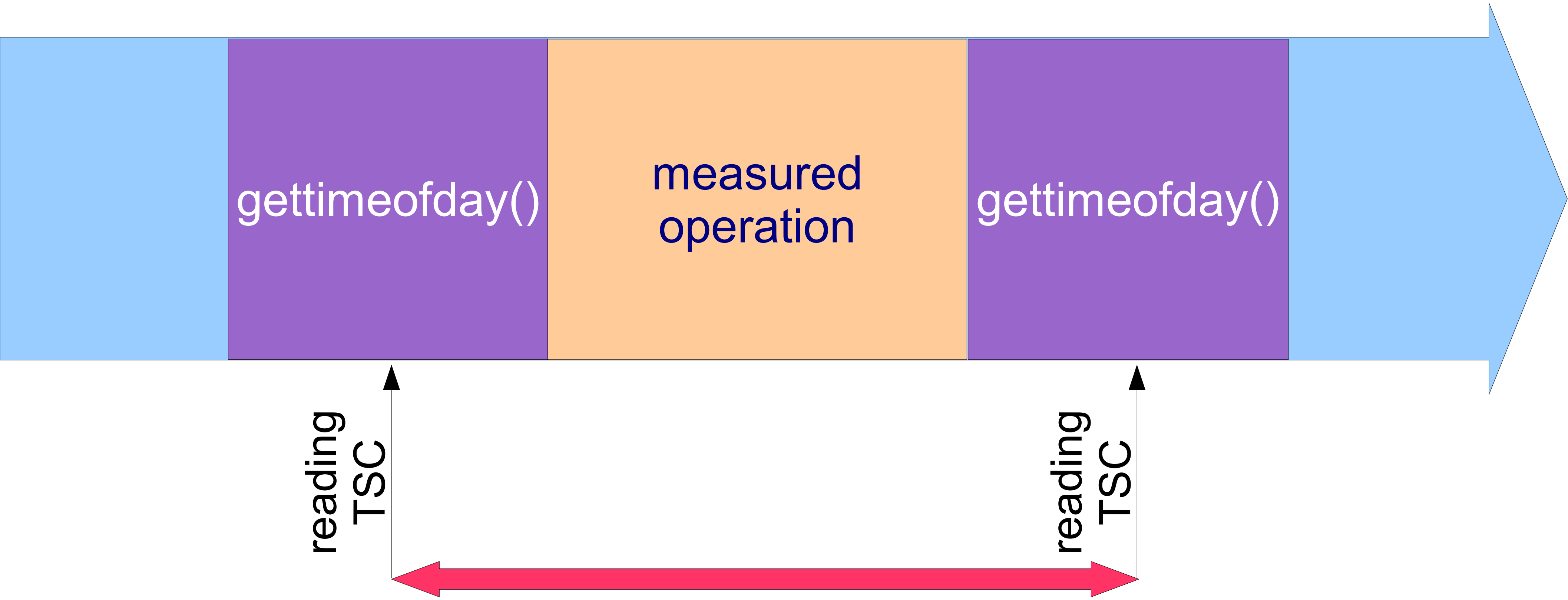
A simple measurement:

```
t1 = gettimeofday();
operation being measured
t2 = gettimeofday();
print "time was ", t2 – t1, "\n";
```

Potential problems:

Inaccuracy due to measurement overhead

measurement overhead:



measured value includes operation + one gettimeofday()

Thus one gettimeofday() should be subtracted

A simple measurement:

```
t1 = gettimeofday();
operation being measured
t2 = gettimeofday();
print "time was ", t2 – t1, "\n";
```

Potential problems:

Inaccuracy due to measurement overhead

Operation is shorter than our measurement resolution (get t1 = t2 so time = 0)

Operation is short relative to random errors (so actually get a random time)

Solution 1: iterate making multiple measurements

```
for (i=0 ; i<N ; i++) {
    t1 = gettimeofday();
    operation being measured
    t2 = gettimeofday();
    print "time was ", t2 – t1, "\n";
}
```

Resolution problem: printouts will be 0 or 1

Random errors: printouts will fluctuate wildly

But in both cases the average (for large enough N) should be good

Solution 1a: buffer the results to reduce measurement interference

```
for (i=0 ; i<N ; i++) {
    t1 = gettimeofday();
    operation being measured
    t2 = gettimeofday();
    time[i] = t2 - t1;
}
print "average is ", avg(time[0..N-1]), "\n";
```

Important because printing is a heavy operation that affects cache state and may cause a context switch

Solution 2: invert loop and measurement

```
t1 = gettimeofday();
for (i=0 ; i<N ; i++) {
    operation being measured
}
t2 = gettimeofday();
print "average was ", (t2 – t1)/N, "\n";
```

Measurement of t2 – t1 will now be stable and meaningful

But need to subtract loop overhead (found by measuring an empty loop)

Problem: compiler optimizations

Compiler may optimize away an empty loop

Even if it has an operation in it like *j*++

If *j* is not subsequently used, why update it?

Even if it is, it's value will grow just like that of the loop index

Could work if the operation is on a global variable, because cross-procedural optimization is harder

Still, need to check

e.g. verify that measured overhead depends on number of iterations

Solution 2a: reduce effect of overhead by unrolling

```
t1 = gettimeofday();
for (i=0 ; i<N/3 ; i++) {
    operation being measured
    operation being measured
    operation being measured
}
t2 = gettimeofday();
print "average was ", (t2 – t1)/N, "\n";
```

Need to decide number of repetitions

In all iterative measurements, need to define N

N should be big enough to pass resolution limit and average out random errors

N should not be too big so as to reduce risk of large interferences from other activities in the system

Underlying assumption that measurement is short

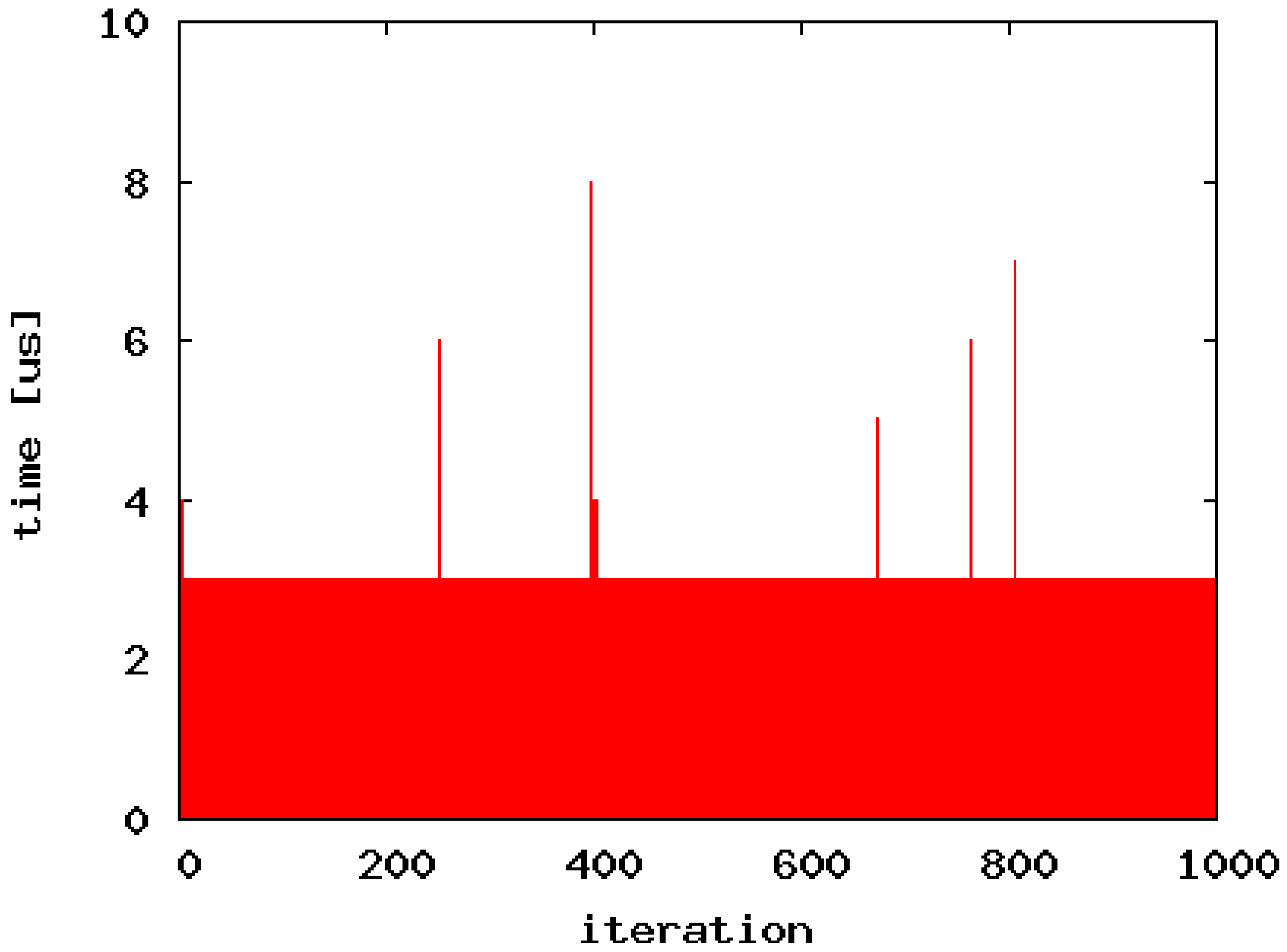
Therefore completed within a single scheduling quantum without interference

If this is not the case, need to account for time spent doing other things

But what if we're unlucky?

Example: time to write 100 bytes at the beginning of a file

outliers are not common, but do happen



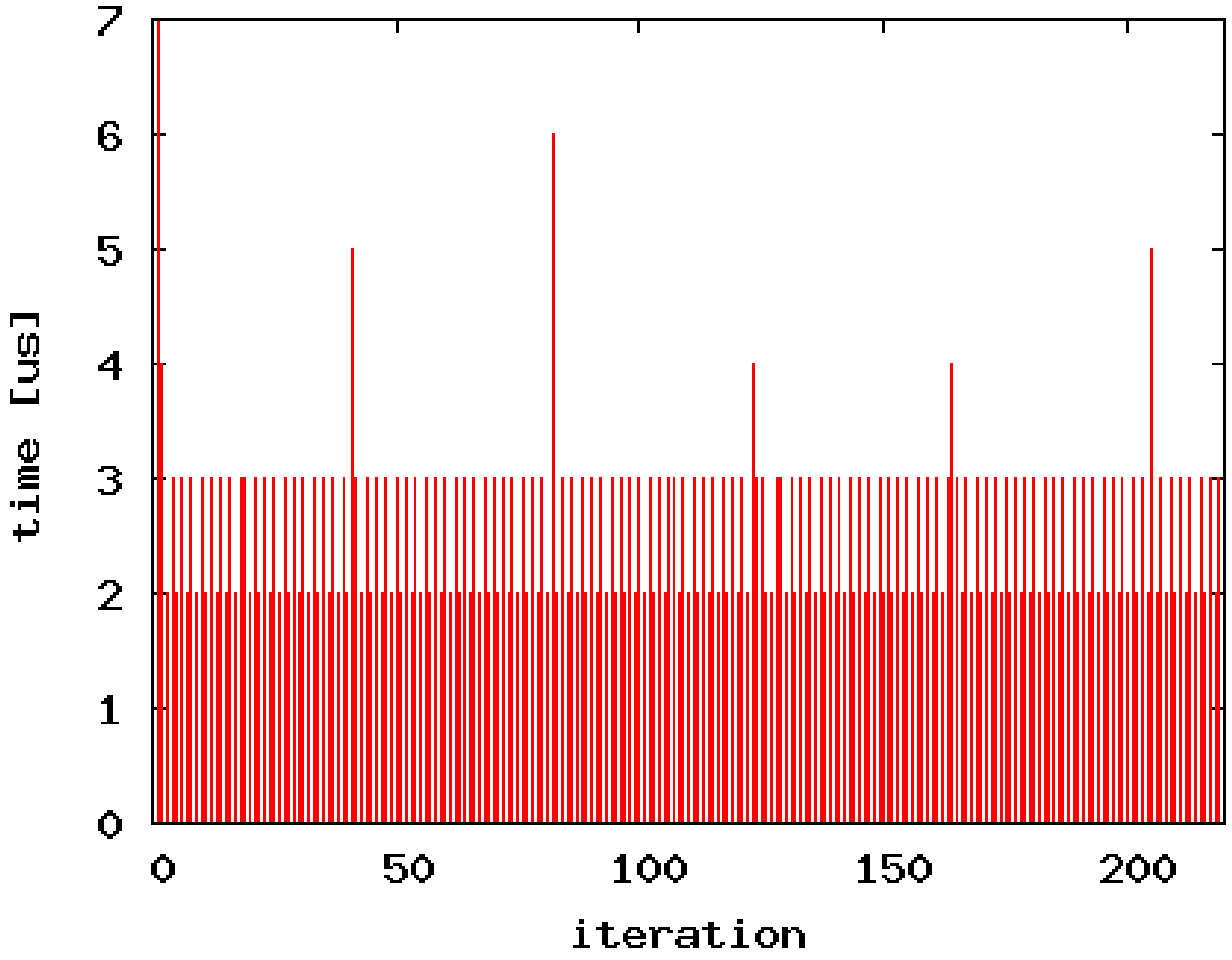
Solution 3: double loop

```
for (r=0 ; r<REP ; r++) {
    t1 = gettimeofday();
    for (i=0 ; i<N ; i++) {
        operation being measured
    }
    t2 = gettimeofday();
    print "average was ", (t2 – t1)/N, "\n";
}
```

Catch outliers in which interference has occurred

Outliers may also exhibit periodic behavior indicative of some system activity that needs to be understood

Example is writing successive blocks of 100 bytes to a file



Biggest problem:

Systematic errors that don't make sense

Need to look at the data

    notice that something is wrong

    identify it precisely

    remove it from the analysis

Hard to do if sensitive to exact conditions

example
code:

```
for (i=0 ; i<=10 ; i++) {
    gettimeofday( &ts[i], 0 );
}
for (i=1 ; i<=10 ; i++) {
    printf("delta=%d\n",
            ts[i].tv_usec - ts[i-1].tv_usec);
}

gettimeofday( &ts[0], 0 );
gettimeofday( &ts[1], 0 );
...
gettimeofday( &ts[10], 0 );

for (i=1 ; i<=10 ; i++) {
    printf("delta=%d\n",
            ts[i].tv_usec - ts[i-1].tv_usec);
}
```

Possible output:

```
delta=1
delta=1
delta=1
delta=1
delta=1
delta=0
delta=1
delta=1
delta=1
delta=1

delta=2
delta=1
delta=1
delta=1
delta=1
delta=1
delta=1
delta=1
delta=1
delta=1
```

But also:

```
delta=1
delta=1
delta=1
delta=1
delta=0
delta=1
delta=1
delta=1
delta=1
delta=1

delta=103
delta=1
delta=1
delta=33
delta=1
delta=1
delta=1
delta=1
delta=1
delta=1
```

so we seem to have some random interference

But if we repeat this 10 times and average, a typical result is

```
avg delta = 1.20
avg delta = 1.00
avg delta = 0.70
avg delta = 0.90
avg delta = 1.00
avg delta = 1.00
avg delta = 0.90
avg delta = 1.00
avg delta = 1.10
avg delta = 1.00
```

(note: typical is not always...)

```
avg delta = 11.10
avg delta = 0.90
avg delta = 1.10
avg delta = 0.90
avg delta = 0.90
avg delta = 1.00
avg delta = 0.90
avg delta = 1.00
avg delta = 1.00
avg delta = 0.90
```

so there is actually some systematic problem at the beginning of the second batch of measurements

Sensitivity:

This only happens when output is directed to the terminal; all is well if it is directed to a file

This is related to printing the results of the first batch just before starting the second batch; all is well if all printing is done at the end

Calculating Confidence Intervals

When we perform multiple measurements of the same thing, we can calculate confidence intervals

Assume measurements are samples from a (normal) distribution

Characterize the distribution's dispersion

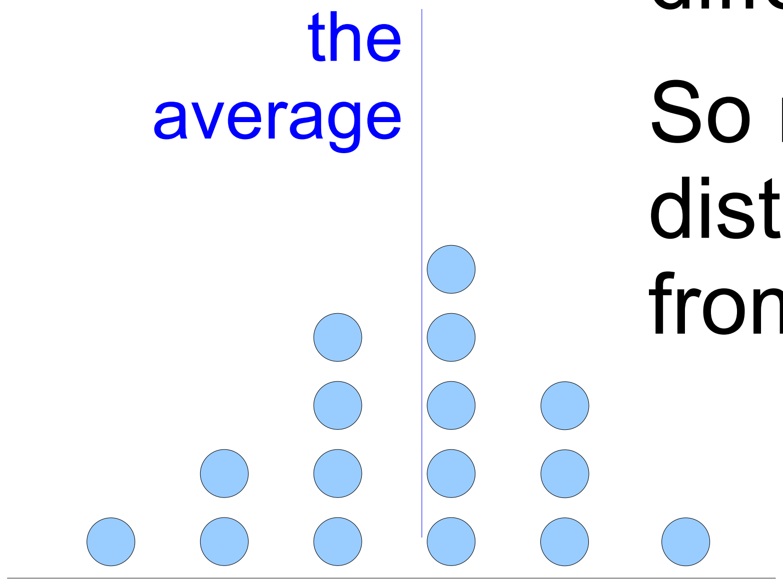
Find the range that includes the desired mass of the probability density (e.g. 90%)

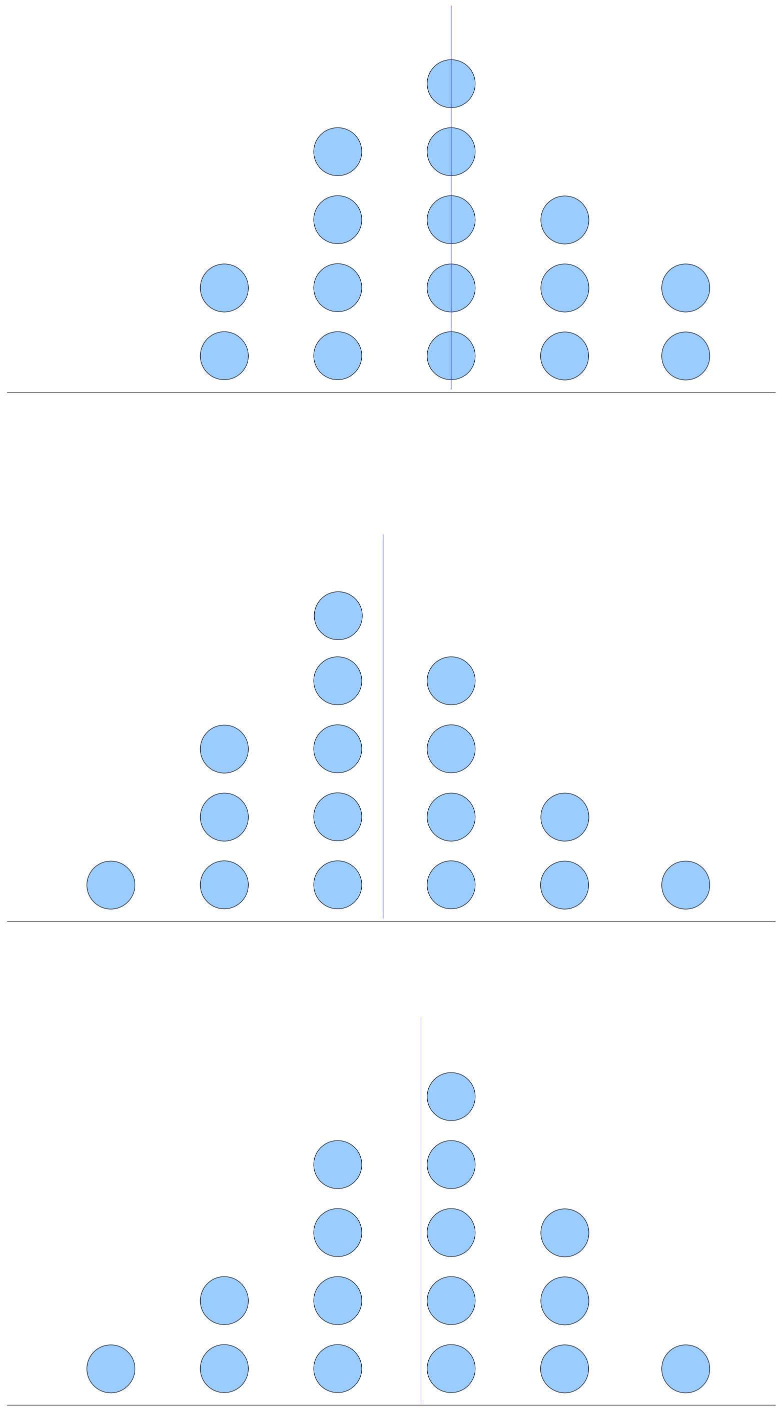
Assume a set of measurements come from a normal distribution (real value + random error)

This set has an average, which is an estimate of the real value

If we repeat this with different samples, we will get a slightly different average

So multiple samples from the base distribution induce a single sample from the distribution of averages

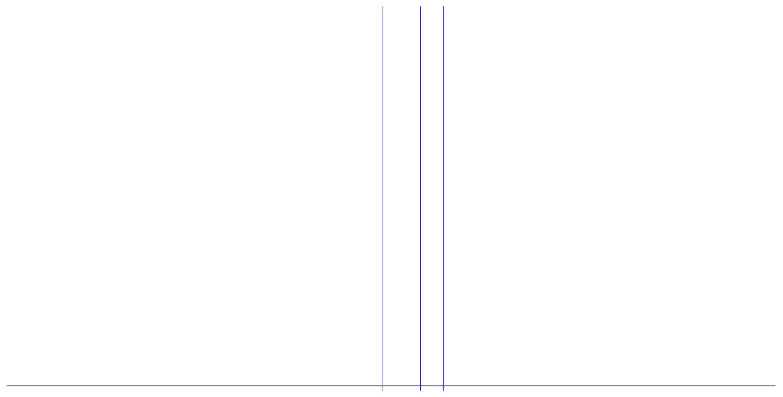




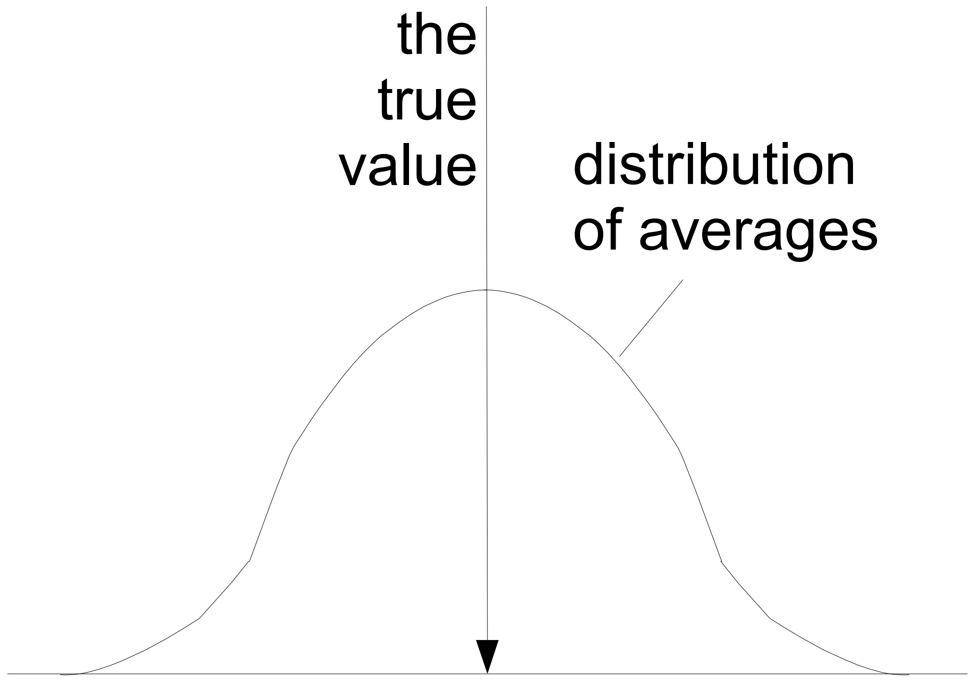
multiple sets of samples induce multiple samples from the distribution of averages

The distribution of averages is narrower than the base distribution

So it gives a tighter estimate of the real value



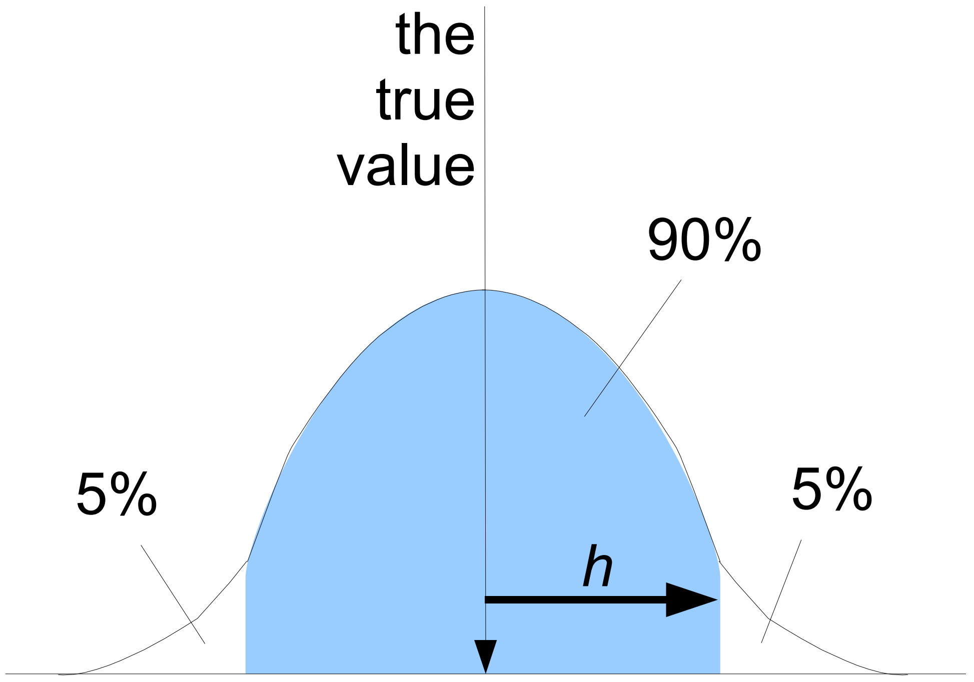
- Assumption: the averages reflect a true value plus some random noise
- Thus the averages are distributed around the true value



Assumption: the averages reflect a true value plus some random noise

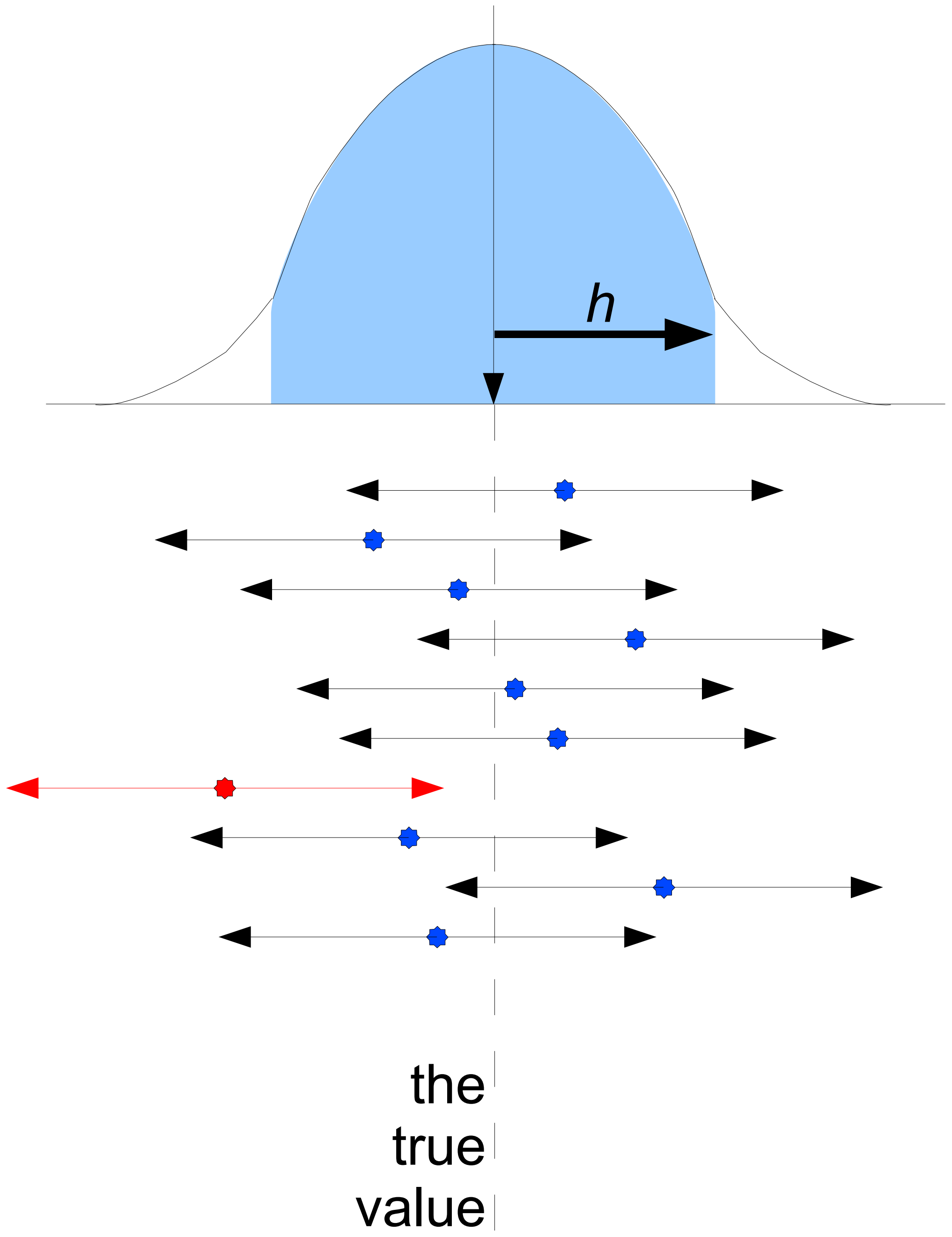
Thus the averages are distributed around the true value

Given the distribution, we can find the range *h* that is expected to contain 90% of the averages



- This also works the other way: for 90% of the averages, the true value is within *h*

- *So the range average ± h has probability 0.9 to include the real value*



Let μ denote the real mean of the base distribution

Let $\bar{x}$ denote the average of *n* samples

If the base distribution is normal, then the averages have a *t* distribution

Let α denote the acceptable uncertainty (implying that the level of confidence is 1-α)

Define the half-width to be

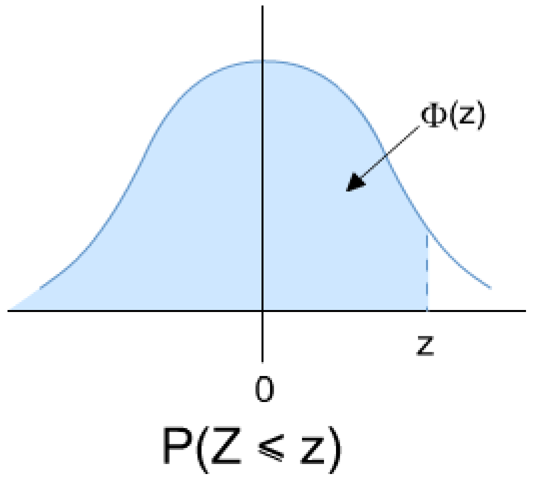
$$h = t_{n-1,1-\alpha/2} s_{\bar{x}}$$

Then

$$p(|\bar{x} - \mu| < h) = 1 - \alpha$$

The half width:

$$h = t_{n-1,1-\alpha/2}\, s_{\bar{x}}$$



$t_{n-1,1-\alpha/2}$ comes from tables

n-1 degrees of freedom

n is the number of samples

for large *n* approximated by the normal $z_{1-\alpha/2}$

$s_{\bar{x}}$ is the standard deviation of the averages

assuming the base samples are *independent*, this can be calculated as $s/\sqrt{n}$ (where *s* is the standard deviation of the base samples)

with more samples, the distribution of the averages becomes narrower

The confidence interval:

$$p(|\bar{x} - \mu| < h) = 1 - \alpha$$

with a certainty of $1 - \alpha$, the distance between a sample of the average $\bar{x}$ and the true mean μ is less than h

If we repeat this many times, and each time we draw a segment of $\pm h$ around $\bar{x}$, then in $1 - \alpha$ of the cases this segment will include μ

Assumptions:

The base samples come from a normal distribution

- If not, but have a finite variance, the averages will still be normal, but this requires a larger *n*

Base samples are independent

- If not, maybe using larger batches will reduce the correlation between them

Note: first clean the data, then compute confidence intervals

Remove outliers that indicate interference

Mechanized approach: remove top and bottom measurements

Better approach: look at the data!

Usually, outliers are only bigger (interference cannot reduce the measured time)

Remove history/warmup effects