Experimental Approaches in Computer Science

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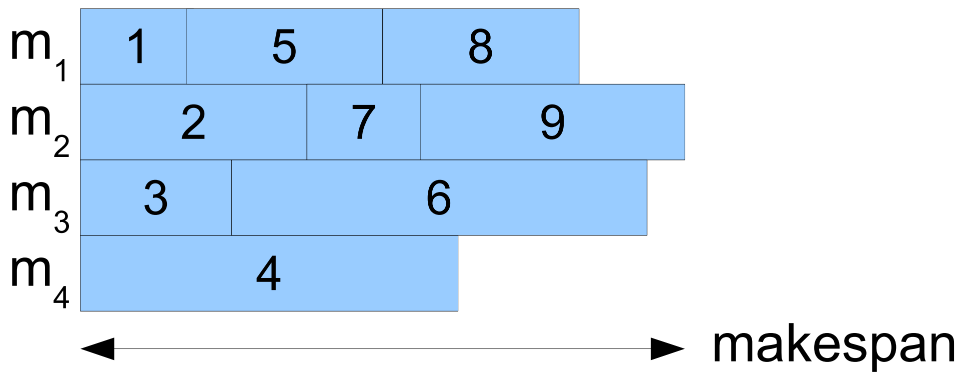
Lecture 12 – Experimental Algorithmics

Case studies

- Online scheduling
- Matrix multiplication
- Maximum flow

Online scheduling

- Problem definition: Given *n* jobs with known processing times assign them to *m* identical machines so as to minimize the makespan



Online: assign each job before you know about subsequent jobs

- Graham's list scheduling [1966]: assign each job to the machine with the least assigned load so far

- Claim: Graham's simple greedy algorithm is $\left(2-\frac{1}{m}\right)$-competitive

Proof:

Let c^* denote the optimal makespan

then $c^* \geq p_{max}$ [accommodate longest job]

and $c^* \geq 1/m \sum p_j$ [accommodate total processing needed]

assume job k is the last one to terminate

then it starts no later than $1/m \sum_{j \neq k} p_j$

because no machine is idle before all jobs start

Its termination time is then no later than its start time + processing time:

$$c_k \leq 1/m \sum_{j \neq k} p_j + p_k$$

$$\leq 1/m \sum_j p_j + (1 - 1/m)p_k$$

$$\leq c^* + (1 - 1/m)c^*$$

$$= (2 - 1/m)c^*$$

Worst case: many small jobs followed by one long job

Improvements:

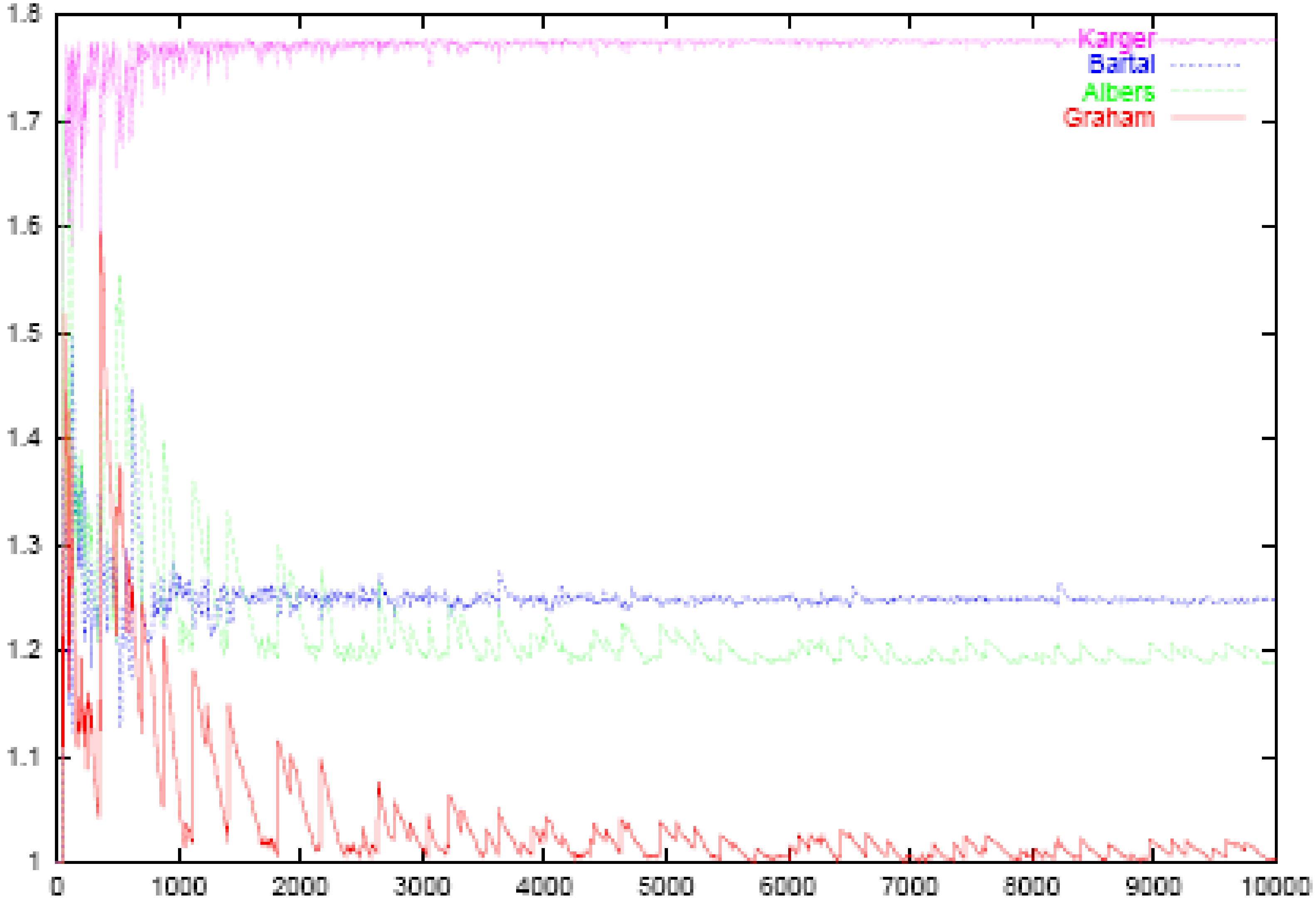
- Bartal et al. [1995]: 1.986-competitive algorithm
- Karger et al. [1996]: 1.945-competitive algorithm
- Albers [1997]: 1.923-competitive algorithm
- All use various conditions to sometimes select a machine that is not the least loaded for short jobs (leaving the least loaded for the long job)
- Question: is this generally good, or does it just avoid certain pathological cases?

Experimental evaluation:

[Albers & Schroder, J. Exp. Alg. 7(3), 2002]

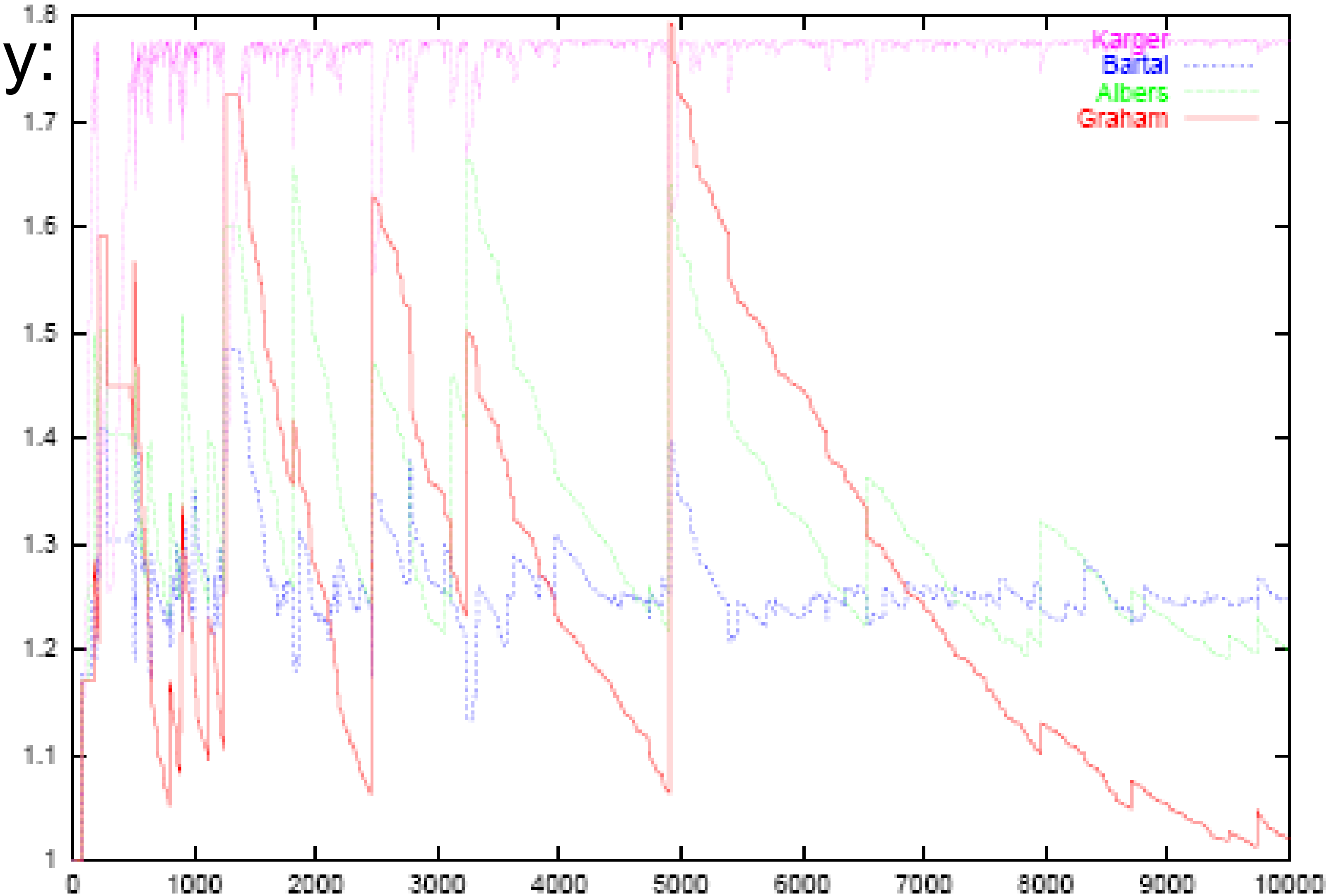
- Use real-world job sizes
  - Parallel machines (MPPs at CTC, KTH)
  - Vector machine (Cray at PSC)
  - Workstation (Sun in Germany)
- Use distributions
- Create sequences of 10000 jobs, and tabulate running ratio of achieved makespan to optimal for m=10

Results KTH:



relatively low variance, so ratio stabilized after some fluctuations; Graham is best

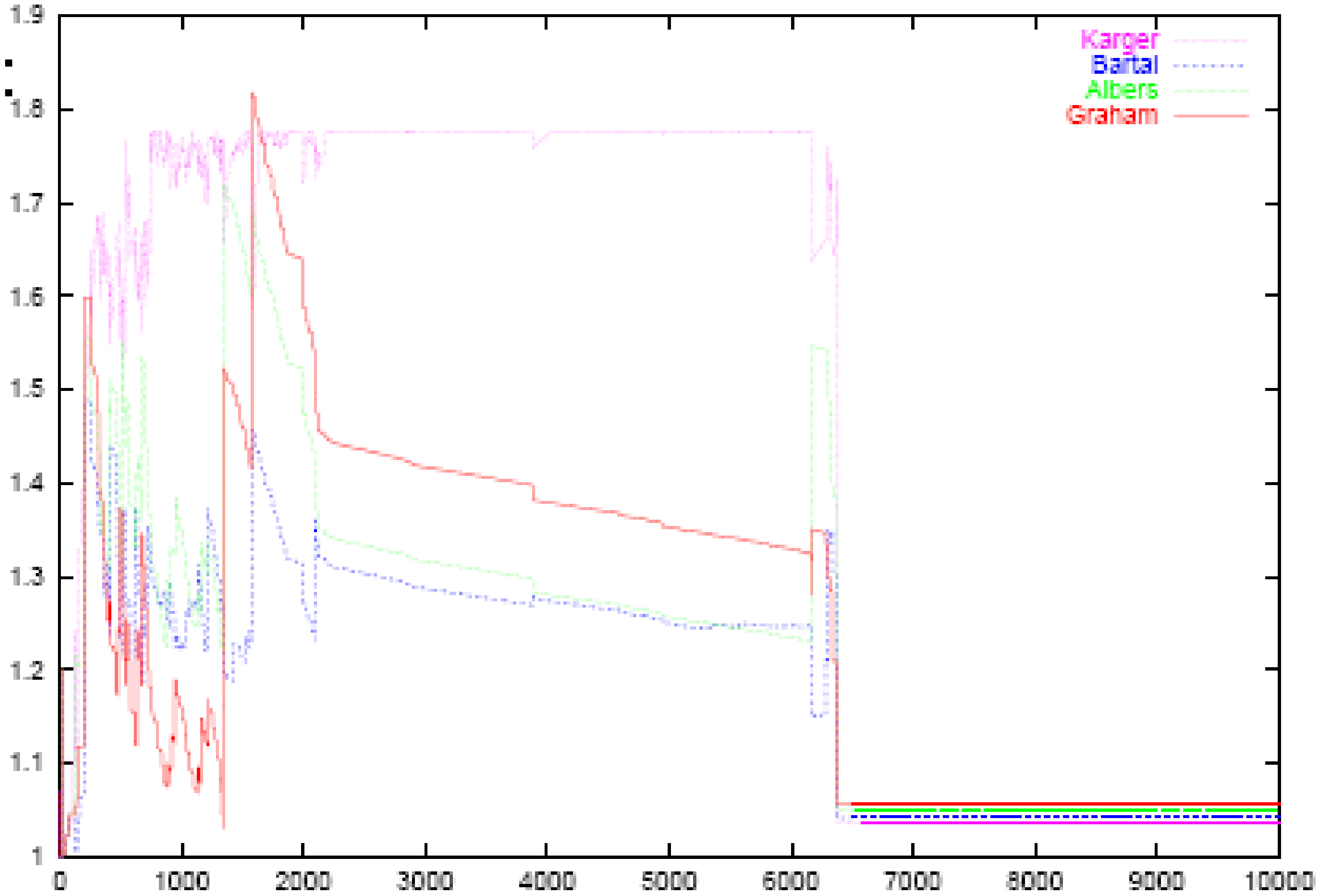
Results Cray:



Occasional big job similar to average so far.

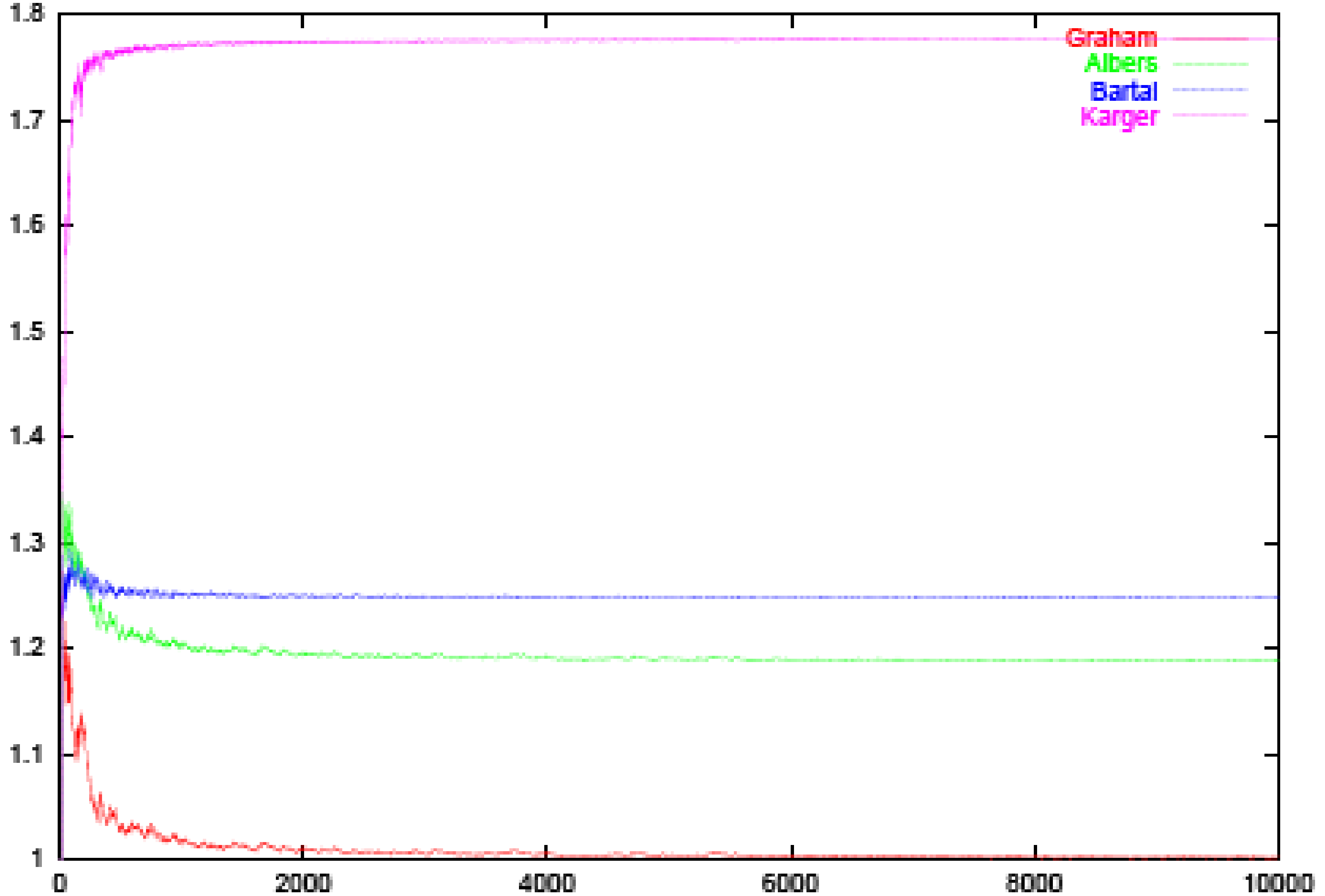
Graham suffers because loads are balanced, and one machine will need to work much more; others leave machines less loaded in anticipation of such jobs

Results Sun:



job sizes have a heavy tail: some are so big they dominate the average. This causes both the online algorithm and the optimal makespan to be essentially equal, and the ratio drops to 1

Exponential:



Relatively low variability leads to quick convergence.

Similar results for uniform, Erlang, and hyperexponential with various parameter values

Effect of number of machines (*m*):

- All previous results were for m=10
- When m grows, it takes longer for ratios to stabilize, because more jobs are needed to fill the machines
- Also, the effect of jobs that are similar to the average load is changed – given that the load is distributed on more machines, these jobs now look huge, and their effect is to reduce the ratio rather than to enlarge it

The bottom line: it depends on the workload

- Graham's simple greedy algorithm is best when job variance is low
- Other algorithms, mainly Albers and Bartal, may reduce sensitivity to large jobs
- When the variance is extremely big due to a heavy tail, the algorithm has little effect

Matrix Multiplication

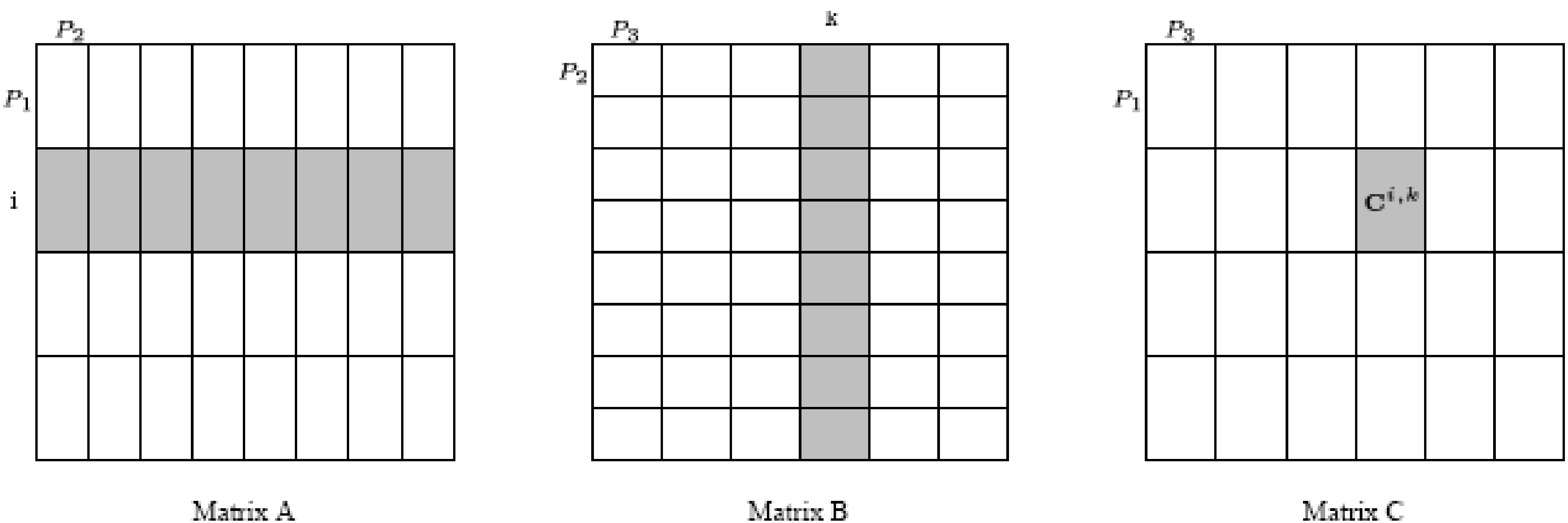
Problem definition:

- Use the straightforward n^3 algorithm
- Take into account the memory hierarchy
  - Cache capacity
  - Cache associativity
  - Contention for the system bus
  - Memory latency
- An instance of algorithm engineering

[Eiron et al. J. Exp. Alg. 4(3), 1999]

Idea 1: use tiling

- Use tiles that fit into the cache, to avoid capacity misses
- Retain ratio of multiple operations per given data



Idea 2: use prefetching

- In each phase prefetch the data needed in the next phase
- If all data is in the cache, computation does not use the system bus at all
- Bus is therefore free for use by prefetching
- Need to time the prefetches so as to avoid evicting needed data (assumes LRU cache replacement)

Tile size constraints

- Computation per tile multiplication is $O(P_1P_2P_3)$
- Data to prefetch is $O(P_1P_2+P_2P_3+P_1P_3)$
- Also need to write back C tile of P_1P_3
- Enough time if $P_1P_2P_3 > P_1P_2+P_2P_3+2P_1P_3$
- Enough space if $2(P_1P_2+P_2P_3+P_1P_3) < C$
- Can reduce prefetching/writeback by reusing C tile for full row of A tiles and column of B tiles

Idea 3: copy to avoid conflicts

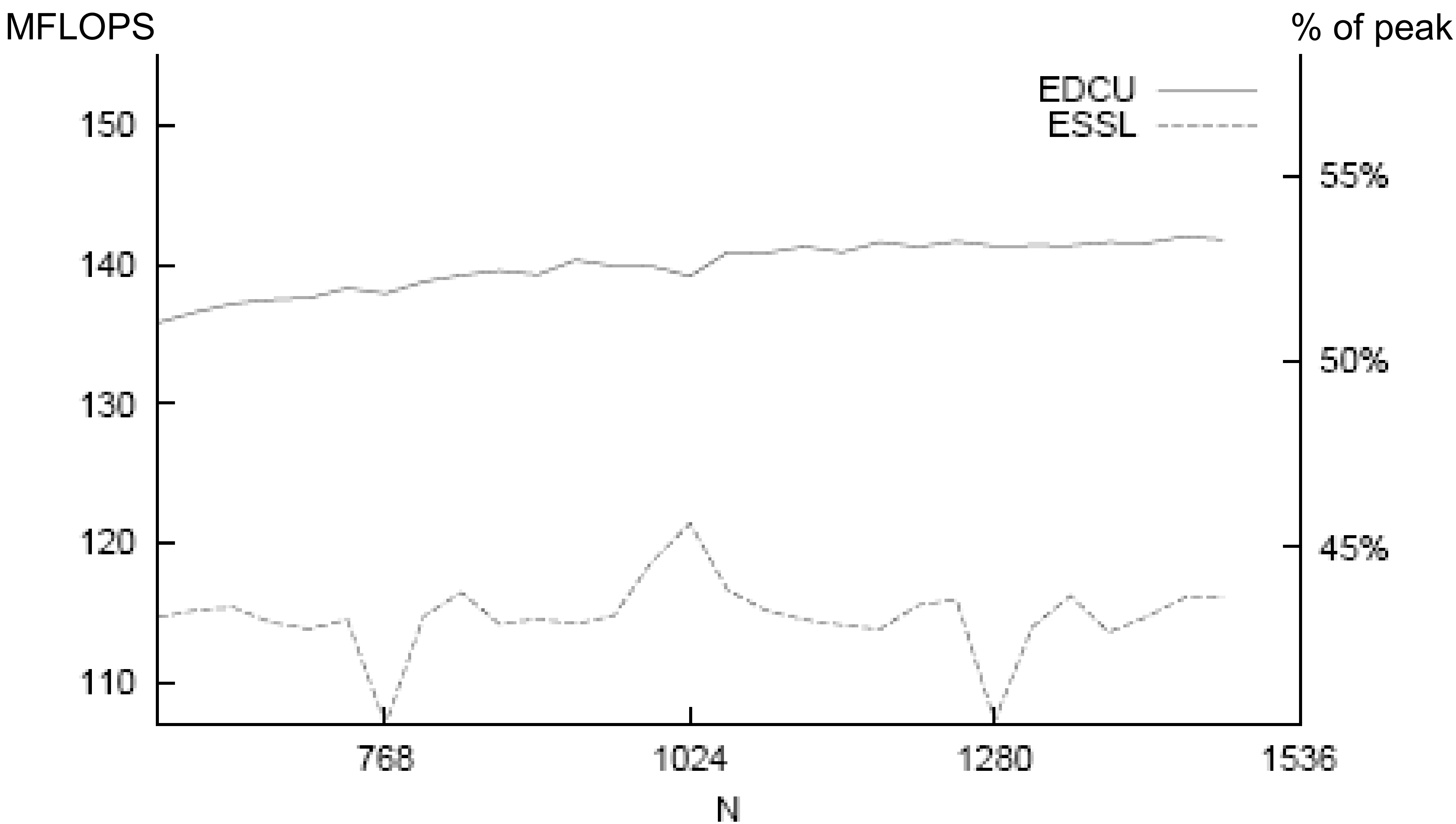
- Copy tiles to different addresses so that they fall in different cache associativity sets
- Assuming *k*-way associativity, ensure that each set is used only *k*/2 times
- Simple example:
  - 2-way associativity
  - Interleave tiles from the different matrices
  - Use offset that is a multiple of the way size
  - Being 2-way allows 2 tiles from each matrix to be cache resident

Implementation:

- IBM PowerPC model 604
- Use fma (floating multiply-add) instruction, which is ideal for matrix/vector multiplication
  - Theoretical peak of 266 MFLOPS
- Don't use dcbt (data cache block touch) instruction for prefetching, but rather a register load
  - dcbt doesn't work when TLB misses
  - Can't be triggered from source level

Performance:

better and more predictable than highly tuned code



Maximum Flow

Problem definition:

given a graph $G=(V,E)$,

with two distinguished nodes s and t,

where each edge e has capacity $c(e)$,

find the maximum possible flow from s to t

we'll focus on unit capacity (c(e)=1 for all edges)

Flow definition:

A flow is a function $f : V \times V \rightarrow R$ such that

- $f(u,v) \leq c(u,v)$ [capacity constraint]
- $f(u,v) = -f(v,u)$ [anti-symmetry]
- $\sum_v f(u,v) = 0$ [conservation constraint]

(holds for all u except s and t)

The value to maximize is $\sum_v f(s,v)$

Main algorithms:

- Path augmentation
- Preflow push-relabel

Path augmentation

- Invariant: always maintain a legitimate flow
- Start with a 0 flow
- At each step
  - Find a path from *s* to *t* that has capacity to spare
  - Add a flow along this path
- Terminate when no additional paths can be found
- Complexity: O(E |f|) with integer capacities, |f| is max

Variants:
BFS? DFS?

Preflow push-relabel

- Invariant: maintains a preflow (allow excess input to a node)
- Initially *s* is at level |V|, *t* and all others at 0
- For all overflowing nodes (starting with *s)* fill outgoing links to nodes at lower level to capacity
- If all unsaturated outbound links are to nodes at same or higher level, relabel the node to level one higher than lowest unsaturated neighbor
- At end, nodes with excess flow will migrate to above the source and push th
- Complexity: O(V^2 E)

Variants: order of push and relabel ops, use of optimizations

Optimizations:

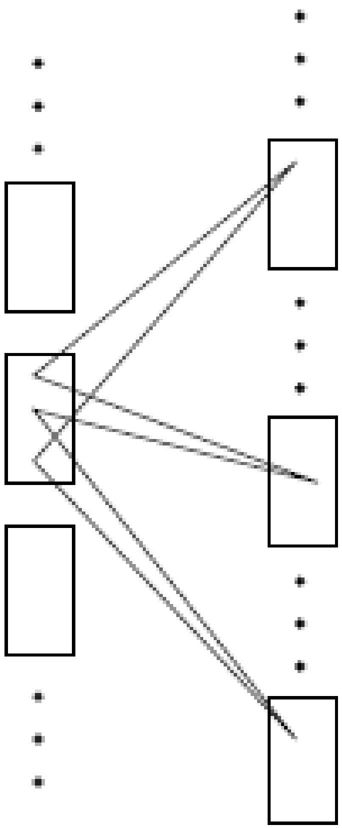
- Global relabel
  - Push and relabel are local operations
  - State may drift away from global optimum
  - Optimization is to do a global scan and relabel all nodes consistently in one sweep
- Gap heuristic:
  - If there are no nodes with label *d*, all those with higher labels return excess to *s*
  - Saves the need to raise their level by single steps to above |V|

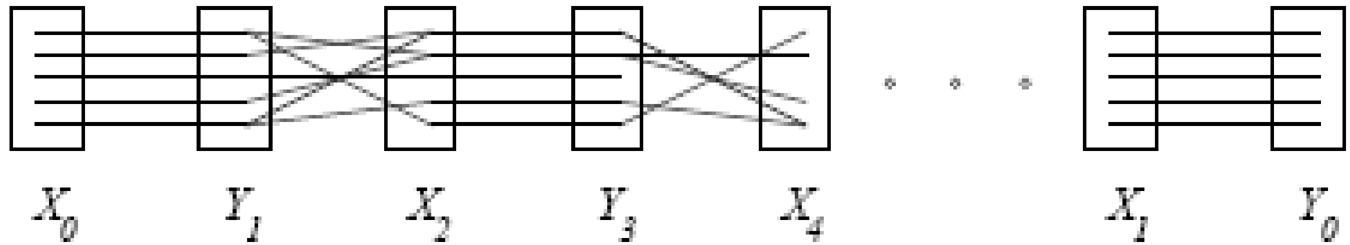
Experimental questions:

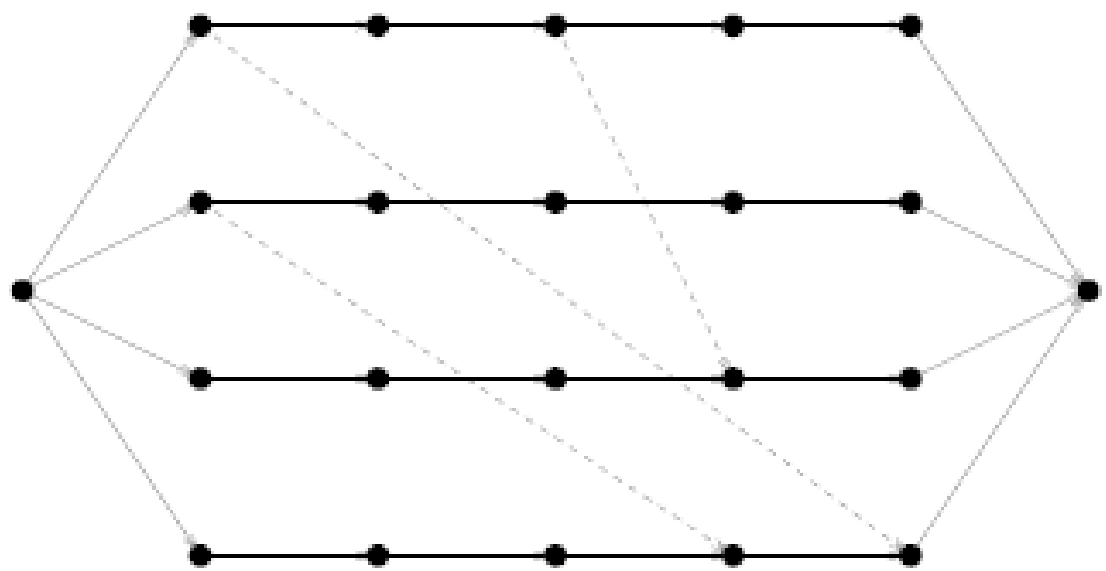
- Augment or push?
- What is the effect of variants and optimizations?
- How does this depend on different input graph instances?

[Cerkassky et al. J Exp. Alg. 3(8), 1998]

Methodology: use random graphs from various different families







Experimental results

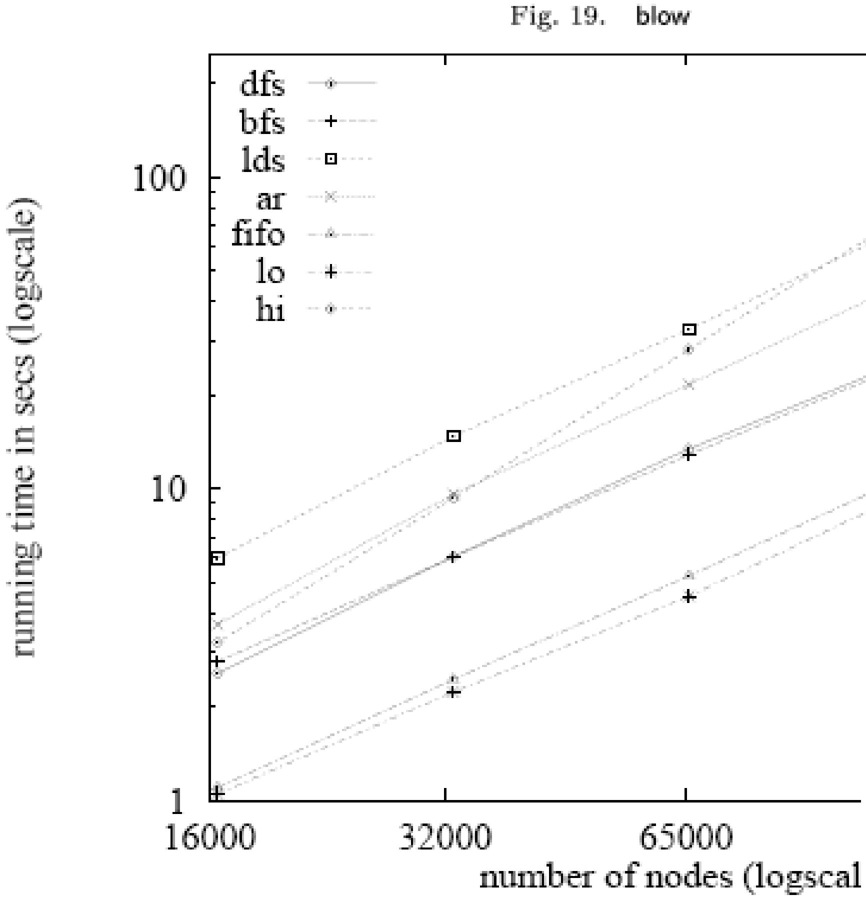
Table 1. Summary of results. *Blank* is good, ○ is fair, and ● is poor.

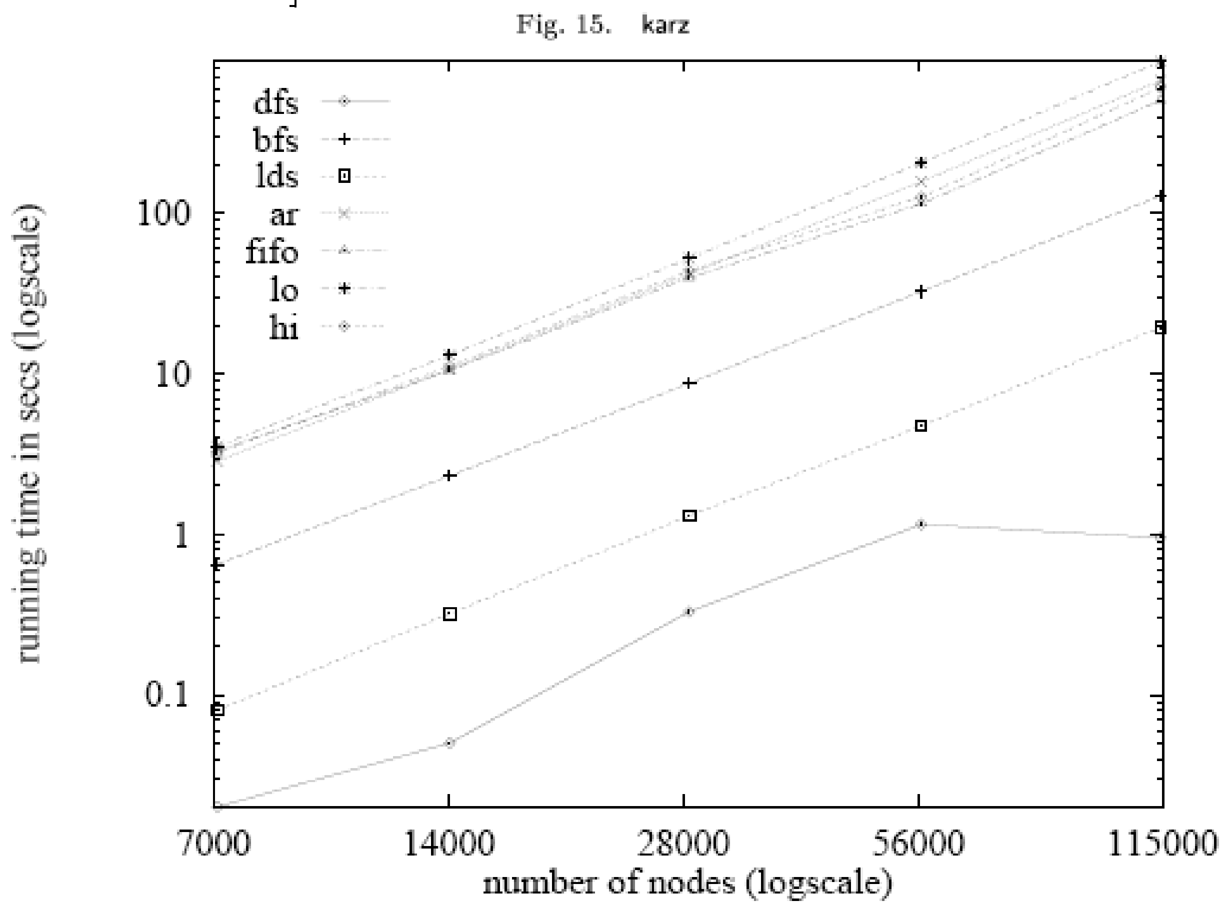
	DFS	BFS	LDS	AR	FIFO	LO	HI
fewg	●	○					
manyg	●	○					
hi-lo							●
grid	●						
hexa	●	○					
rope							○
zipf					○		
karz		●	○	●	●	●	●
rmfuC	●	●	○	○			
rmfuL	○	○		○			
rmfuW	●	●	○	○			
blow			○	○			○
puff	○	●		○			○
saus					●	●	●
squa							○
wave	●	●		○			○

Rows are families of graphs

columns are algorithms

Experimental results





Plots for graph families
Lines for algorithms

Conclusions:

- No single algorithm is best for all graph types
- Both BFS and DFS (path augmentation) are not robust, with bad performance for many graph families
- The best push-relabel methods are generally more robust than the best augmented flow
- The added heuristics are important for the achieved performance