Social Distance Games

(Extended Abstract)

Simina Brânzei
Cheriton School of Computer Science
University of Waterloo
Waterloo, Canada
sbranzei@uwaterloo.ca

Kate Larson
Cheriton School of Computer Science
University of Waterloo
Waterloo, Canada
klarson@uwaterloo.ca

ABSTRACT

In this paper we introduce and analyze *social distance games*, a family of non-transferable utility coalitional games where an agent's utility is a measure of closeness to the other members of the coalition. We study both social welfare maximisation and stability in these games from a graph theoretic perspective. We investigate the welfare of stable coalition structures, and propose two new solution concepts with improved welfare guarantees. We argue that social distance games are both interesting in themselves, as well as in the context of social networks.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence - *Multiagent Systems*; J.4 [**Computer Applications**]: Social and Behavioral Sciences - *Economics*

General Terms

Theory, Economics, Algorithms

Keywords

Social networks, Cooperative games, Solution concepts, Core

1. INTRODUCTION

In recent years, there has been growing interest in the game theoretic analysis of social and economic network formation ([3]). Social networks play a crucial role in everyday life and influence all aspects of behaviour, such as where people live and work, what music they listen to, and with whom they interact. Early work on social networks was done by Milgram in the 1960's and his experiments suggested that any two people in the world are connected by a path of average length six. Since then, researchers observed that many natural networks, such as the web, biological networks, networks of scientific collaboration, exhibit the same properties as the web of human acquaintances.

In this paper we present a novel coalitional game that models the interaction of agents in social networks using the notion of social distance. Our game captures the idea that agents in a social network receive utility from maintaining ties to other agents that are close to them, but have to pay for maintaining distant ties. Using social distance games, we study the properties of efficient and stable networks, relate them to the underlying graphical structure of the game, give an approximation algorithm for finding optimal social welfare, and propose two solution concepts with improved welfare guarantees.

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2. THE MODEL

Definition 1 *A* social distance game *is represented as a simple unweighted graph* $G = (N, E)$ *where*

- $N = \{x_1, \ldots, x_n\}$ *is the set of agents*
- *The utility of an agent* x_i *in coalition* $C \subseteq N$ *is*

$$u(x_i, C) = \frac{1}{|C|} \sum_{x_j \in C \setminus \{x_i\}} \frac{1}{d_C(x_i, x_j)}$$

where $d_C(x_i, x_j)$ *is the shortest path distance between* x_i *and* x_j *in the subgraph induced by coalition* C *on* G. *If* x_i *and* x_j *are not connected in* C, $d_C(x_i, x_j) = \infty$.

Our utility formulation is a variant of closeness centrality, is well defined on disconnected sets, and normalized in the interval $[0, 1]$. It is also related to classical measures used in graph theory network analysis, such as degree centrality, closeness centrality, and betweenness centrality. Let a coalition structure, P, be a partition of the set of agents into coalitions. The set of agents, N, is also known as the *grand coalition*, and we denote its size by $|N| = n$.

Definition 2 *The* social welfare *of coalition structure* $P = (C_1, \ldots, C_k)$ *is*

$$SW(P) = \sum_{i=1}^{k} \sum_{x_j \in C_i} u(x_j, C_i)$$

We sometimes refer to the utility of agent x_i in partition P as $u(x_i, P)$ or, when the context is clear, as $u(x_i)$.

The main notion of stability that we study in this paper is the *core* solution concept.

Definition 3 *A coalition structure* $P = (C_1, \ldots, C_k)$ *is in the* core *if there is no coalition* $B \subseteq N$ *such that* $\forall x \in B$, $u(B, x) \geq u(P, x)$ *and for some* $y \in B$ *the inequality is strict.*

If coalition structure P is in the core, P is resistant against group deviations. No coalition B can deviate and improve at least one member, while not degrading the others. If B exists, it is called a *blocking coalition*.

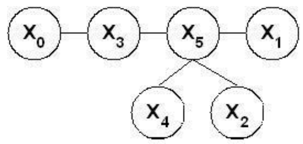

Figure 1: In $\{x_0, x_1, x_2, x_3, x_4, x_5\}$, $u(x_0) = \frac{1}{5}(1 + 1/2 + 3 \cdot 1/3) = \frac{1}{2}$, $u(x_5) = \frac{1}{5}(1/2 + 4 \cdot 1) = \frac{9}{10}$. In $(\{x_0, x_3\}, \{x_1, x_2, x_4, x_5\})$, $u(x_0) = u(x_3) = \frac{1}{2}$, $u(x_1) = \frac{1}{2}$, $u(x_2) = u(x_4) = \frac{1}{2}$, $u(x_5) = \frac{3}{4}$.

3. SOCIAL WELFARE

In this section we give an $O(n)$ algorithm to approximate optimal welfare within a factor of two. The algorithm decomposes the graph into non-singleton connected components, such that each component has diameter at most two. We call this type of partition a diameter two decomposition.

Theorem 1 *Diameter two decompositions guarantee to each agent utility at least* $1/2$.

The diameter two decomposition is an approximation of optimal welfare that satisfies at the same time a notion of *fairness*: *every* agent is guaranteed to receive more than half of their best possible value.

Algorithm 1 Fair Approximation of Optimal Welfare

$T \leftarrow$ Minimum-Spanning-Tree(G);
$k \leftarrow 1$;
while $|T| \geq 2$ **do**
  $x_k \leftarrow$ Deepest-Leaf(T);
  $C_k \leftarrow \{\text{Parent}(x_k)\}$;
  for all $y \in$ Children(Parent(x_k)) **do**
    $C_k \leftarrow C_k \cup \{y\}$;
  end for
  // Remove vertices C_k and their edges from T
  $T \leftarrow T - C_k$;
  $k \leftarrow k + 1$;
end while
// If the root is left, add it to the current coalition
if $|T| = 1$ **then**
  $C_k \leftarrow C_k \cup \{\text{Root}(T)\}$;
end if
return $(C_1, \ldots, C_k)$;

4. THE CORE

Group stability is an important concept in coalitional games. No matter how many desirable properties a coalition structure satisfies, if there exist groups of agents that can deviate and improve their utility by doing so, then that configuration can be easily undermined. There exist social distance games with empty cores (Figure 1). The grand coalition is blocked by $\{x_1, x_2, x_4, x_5\}$, partition $(\{x_0, x_3\}, \{x_1, x_2, x_4, x_5\})$ is blocked by $\{x_1, x_2, x_3, x_4, x_5\}$.

4.1 Core Stable Partitions are Small Worlds

A small world network is a graph in which most nodes can be reached from any other node using a small number of steps through intermediate nodes. The expected diameter of small world networks is $O(\ln(n))$. Most real networks display the small world property, and examples range from genetic and neural networks to the world wide web [1]. In this model, core stable partitions divide the agents into small world coalitions, regardless of how wide the original graph was. We obtained an upper bound of 14 on the diameter of any coalition in the core.

Theorem 2 *The diameter of any coalition belonging to a core partition is bounded by the constant* 14.

5. STABILITY GAP

We analyse the loss of welfare that comes from being in the core using the notion of stability gap [2], which is the ratio between the best possible welfare and the welfare of a core stable partition (if it exists).

Theorem 3 *Let $G = (N, E)$ be a game with nonempty core. Then $Gap_{\min}(G)$ is in worst case $\Theta(\sqrt{n})$.*

6. ALTERNATIVE SOLUTION CONCEPTS

In this section we consider several variations of the core that offer better social support.

6.1 Stability Threshold

The stability threshold is descriptive of situations where agents naturally stop seeking improvements once they achieved a minimum value. This is a well-known assumption observed experimentally as a form of bounded rationality: choosing outcomes which might not be optimal, but will make the agents sufficiently happy.

We analyse stability for a threshold of $k/(k + 1)$, which is equivalent to an agent forming a coalition with k of his direct neighbours. In this case, there can be at most $k - 1$ singletons neighbouring any agent with utility at least $1/2$ in the core, since otherwise the singletons can block with that agent.

Theorem 4 *Let $G = (N, E)$ be an induced subgraph game with nonempty core of threshold $k/(k+1)$. Then $Gap_{\min}(G) \leq 4$ if $k = 1$, and $Gap_{\min}(G) \leq 2k$ if $k \geq 2$.*

6.2 The "No Man Left Behind" Policy

Here we view the formation of core stable structures as a process that starts from the grand coalition and stabilizes through rounds of coalitions splitting and merging. While in general, the search for the core can begin from any partition, initializing with the grand coalition is natural in many situations. For example, at the beginning of any joint project, a group of people gather to work on it. As the project progresses, they may form subgroups based on the compatibilities and strength of social ties between them. We formulate a simple social rule that agents have to follow when merging or splitting coalitions. That is, whenever a new group forms, it cannot leave behind any agent working alone. We call this rule the "No Man Left Behind" policy. The "No Man Left Behind" code of conduct is well known in the army and refers to the fact that no soldier can be left alone in a mission or abandoned in case of injury.

Theorem 5 *Let G be a game which is stable under the "No Man Left Behind" policy. Then $Gap_{\min}(G) < 4$.*

7. REFERENCES

[1] A.-L. Barabasi and Z. N. Oltvai. Network biology: understanding the cell's functional organization. *Nature Reviews Genetics*, 5:101–113, 2004.

[2] S. Brânzei and K. Larson. Coalitional affinity games and the stability gap. In *IJCAI*, pages 79–84, 2009.

[3] M. Jackson, editor. *Social and Economic Networks.* Princeton University Press, 2008.