Bounded Optimal Team Coordination with Temporal Constraints and Delay Penalties

(Extended Abstract)

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ABSTRACT

We address the problem of optimally assigning spatially distributed tasks to a team of heterogeneous mobile agents in domains with inter-task temporal constraints, such as precedence constraints. Due to delay penalties, satisfying the temporal constraints impacts the overall team cost. We present a mathematical model of the problem, a benchmark anytime bounded optimal solution process, and an analysis of the impact of delay penalties on problem difficulty.

Categories and Subject Descriptors

I.2 [**AI**]: Problem Solving, Control Methods, and Search

General Terms

Algorithms

Keywords

Multiagent planning, Coordination

1. INTRODUCTION

Multi-agent coordination problems span the spectrum from loose coordination, in which agents independently perform their assigned tasks, to tight coordination, where all actions are synchronized. Between these two extremes are many scenarios for which there are interdependencies between the schedules of different agents, arising from inter-task temporal constraints such as precedence or synchronization constraints. Furthermore, the manner in which these inter-task constraints are satisfied may impact the overall team cost, as is the case if there is a cost associated with agent delays needed to ensure that constraints are satisfied. We describe such problems as having *cross-schedule dependencies* [4].

We address task allocation, scheduling and routing for a team of heterogenous mobile agents in such scenarios. In particular, the cross-schedule dependencies we focus on are inter-task precedence constraints and delay penalties.

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Although task allocation, scheduling and routing problems are widely studied in multi-robot coordination and vehicle routing, very little has been done to address such cross-schedule dependencies. Some recent work has begun to incorporate inter-task temporal constraints [2, 3]. However, this work does not consider situations where satisfying these constraints has an impact on the overall team cost, a feature of real-world problems in many domains, and a feature that significantly complicates the coordination problem

2. PROBLEM AND APPROACH

A set of mobile agents, K, is available to perform a collection of tasks. Each multi-agent task can be decomposed into simpler single-agent tasks. Each single-agent task $j \in J$ requires specific agent capabilities and consists of one or more spatially distributed subtasks, $i \in I$. Subtasks of different tasks may be related by temporal constraints, thus creating dependencies between different agents' schedules.

We formulate a *set-partitioning* mixed-integer programming model, with side constraints, for this problem. Key variables and constants in this model are summarized in Table 1, while the model itself appears in Figure 1. A binary variable, x_k^r represents whether a given agent k is assigned to a particular *route* (single-agent plan), r, out of all feasible routes R_k for that agent. Thus, solving the model involves generating feasible routes and assigning values of 0 or 1 to route variables so as to maximize the difference between task rewards and travel and delay costs (Eq. 1). Furthermore, each agent must perform at most one route (2), each task is performed on at most one route (3), and precedence constraints are satisfied (4-5). Due to space limitations, necessary constraints for computing task start and delay times are not shown. Also omitted are additional problem features, such as task time windows. The full model is presented in a technical report [5].

We develop a custom *branch-and-price* [1] algorithm, the details of which are also presented in the technical report, that computes progressively better solutions, with bounds on quality, until it returns a provably optimal solution.

3. EXPERIMENTS AND RESULTS

Our test scenario is one in which individuals with special needs must be sheltered in an emergency. Each client with special needs must be visited by a medical agent and then moved to an emergency shelter by a transportation agent. There is a precedence constraint between the medical visit and the client pickup. Furthermore, there are costs asso-

Table 1: Defined variables and terms

Var.	Definition	Type
x^k_r	Whether agent k performs route r	Binary
d^k_i	Delay time of agent k for subtask i	Real
t_i	Execution start time for subtask i	Real
Term	**Definition**	**Type**
R_k	Feasible routes for agent $k \in K$	Set
P	Pairwise precedence constraints	Set
v_j	Value of completing task j.	Real
c^k_{1r}	Travel cost for route $r \in R_k$	Real
c^k_2	Wait cost per unit time for agent k	Real
π^k_{jr}	Whether task j occurs on route $r \in R_k$	Binary
λ_i	Service duration for subtask i	Real
τ_∞	End of planning horizon	Real
y_j	Whether task j is performed $= \sum_{k \in K} \sum_{r \in R_k} \pi^k_{jr} x^k_r$	Binary

Maximize:

$$\sum_{j \in J} \sum_{k \in K} \sum_{r \in R_k} v_j \pi^k_{jr} x^k_r - \sum_{k \in K} \sum_{r \in R_k} c^k_{1r} x^k_r - \sum_{i \in I} \sum_{k \in K} c^k_2 d^k_i \quad (1)$$

Subject to:

$$\sum_{r \in R_k} x^k_r \leq 1 \quad \forall k \in K \quad (2)$$

$$\sum_{k \in K} \sum_{r \in R_k} \pi^k_{jr} x^k_r \leq 1 \quad \forall j \in J \quad (3)$$

$$y_{task(i)} - y_{task(i')} \leq 0 \quad \forall (i', i) \in P \quad (4)$$

$$t_{i'} - t_i + \lambda_{i'} + \tau_\infty (y_{task(i)} - y_{task(i')}) \leq 0 \quad \forall (i', i) \in P \quad (5)$$

Not shown are constraints ensuring the correct computation of the t_i and d^k_i variables. The full model appears in [5].

Figure 1: Key aspects of mathematical model

ciated with agent travel and delay time. Thus, the problem requires joint coordination of transportation and medical agents, considering cross-schedule dependencies.

We focus on two interesting results: first, the anytime, bounded optimal nature of the algorithm, and second, the impact that including delay penalties has on problem difficulty. In the discussion below, a delay penalty of 0 indicates that only travel time is minimized. A delay penalty of 0.5 means that a weighted sum of travel and delay time is minimized, with delay time weighted half as much as travel time.

Figure 2 (left) shows the best solution and best bound over time for an example problem with 6 clients, 1 medical agent, 2 transportation agents, and a delay penalty of 0.5. The algorithm is able to compute progressively better solutions and bounds. Furthermore, it finds good solutions early, but takes longer to prove the optimality of these solutions.

Figure 2 (right) shows the total time to find and prove the optimal solution, averaged over 5 random instances of problem configurations with 1 medical agent, 2 transportation agents, and between 2 and 10 clients. The combinatorial nature of the problem is apparent in the rapid increase in the time needed to prove solution optimality as the problem size increases. Planning time was capped at 30 minutes, and the bottom graph indicates the ratio of the terminating solution to the terminating bound. A ratio of 1 indicates optimality. The figure also highlights the impact of delay penalties on problem difficulty. It illustrates that in the presence of precedence constraints, problems that optimize a weighted sum of travel and delay time are significantly more difficult than problems that optimize travel time alone. This is because the algorithm must essentially evaluate the trade-off between travel time and delay time in potential solutions it encounters during the solution process.

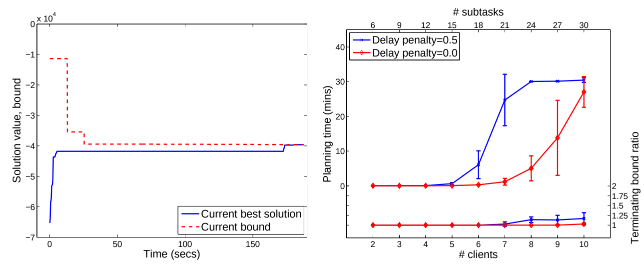

Figure 2: Example solution profile (left) and overall planning time (right)

4. CONCLUSIONS

We present a novel mathematical formulation and anytime bounded optimal solution approach to heterogeneous team coordination with precedence constraints and delay penalties. Our follow-on work addresses additional types of cross-schedule dependencies.

5. ACKNOWLEDGEMENTS

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