Two Points Fundamental Matrix

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Abstract

It is well known that computing the fundamental matrix of two uncalibrated cameras requires at least seven corresponding points. We present a method to compute the fundamental matrix between two cameras with only two pairs of corresponding points. Given these two points, we show how to find three pairs of corresponding epipolar lines, from which the fundamental matrix can be computed.

Two pairs of epipolar lines, incident to the two pairs of corresponding points, are found by maximizing stereo consistency between lines; corresponding epipolar lines yield a good stereo correspondence. These two epipolar lines intersect at the epipoles, giving a third pair of corresponding points. A third pair of matching epipolar lines, needed to compute the fundamental matrix, is found from lines incident to the epipoles.

We validate our method using real-world images and compare it to state-of-the-art methods. Our approach is more accurate by a factor of five compared to the standard method using seven corresponding points, and its accuracy is comparable to the 8-points algorithm.

The proposed two-point algorithm needs much fewer trials when candidate point correspondences also include erroneous matches. The probability to sample two correct correspondences, as needed for our method, is much higher than the probability to sample seven or eight correct correspondences.

1 Introduction

The fundamental matrix is a basic building block of multiple view geometry and its computation is the first step in many vision tasks. The computation is usually based on pairs of corresponding points. Matching points across images is error prone and many subsets of points need to be sampled until a good solution is found. In this work we address the problem of robustly estimating the fundamental matrix from only two corresponding pairs of points.

The best-known algorithm, adapted for the case of fundamental matrix, is the eight points algorithm by Longuet-Higgins [1]. It was made practical by Hartley [2]. The overall method is based on normalization of the data, solving a set of linear equations and enforcing the rank 2 constraint [3]. The requirement of *eight* point correspondences can be relaxed to seven. This results in a cubic equation with one or three real solutions. The estimation from 7 points is very sensitive to noise. These methods are often followed by a non-linear optimization step.

The fundamental matrix can also be computed from three matching epipolar lines [4]. Given three such correspondences, the one dimensional homography between the lines can be recovered and the epipolar lines in each of the images intersect at the epipoles. The three degrees of freedom for the 1D homography together with the four degrees of freedom of the epipoles yield the required 7 degrees of freedom needed to compute the fundamental matrix. The main challenge of such methods is finding corresponding epipolar lines. While most image points have a correspondence in the other image, most lines in the image are *not* even epipolar lines.

We present a method to robustly compute the fundamental matrix from only two corresponding pairs of points. This is achieved by the following steps: (a) Finding corresponding epipolar lines incident to the points based on the brightness consistency (stereo matching) that exists between corresponding epipolar lines. (b) Estimating the fundamental matrix based on three corresponding epipolar lines instead of seven or eight corresponding points.

In all approaches, computing the fundamental matrix from points correspondences starts with the computation of corresponding points, which normally has many outliers. Using a RANSAC approach, multiple subsets of points are sampled, hoping that one selected subset will not include any outlier. Using only 2 points as needed in our method substantially reduces the number of samples. For example, if 50% of correspondences are outliers, and we require a probability of 99% that one subset will have no outliers, we will need to select 1,177 8-point subsets, 588 7-point subsets, and only 17 subsets of 2-points.

2 Previous Work

2.1 Computing the Fundamental Matrix

The fundamental matrix is a 3×3 homogeneous rank two matrix, with seven degrees of freedom. There are various formulations that have been considered to produce a minimal parameterization with only seven parameters [4].

The most common parameterization is from the correspondences of seven points and can be computed as the null space of a 7×9 matrix. The rank two constraint leads to a cubic equation with one or three possible solutions.

The method we will follow is based directly on the epipolar geometry entities. The fundamental matrix is represented by the epipoles and the epipolar line homography. Each of the two epipoles accounts for two parameters. The epipolar line homography represents the 1D-line homography between the epipolar pencils and accounts for three degrees of freedom.

2.2 Stereo Matching Along Epipolar Lines

Depth from two stereo images is traditionally computed by matching along corresponding epipolar lines. Our conjecture is that stereo matching will be more successful when applied to corresponding epipolar lines, rather than to random, unrelated lines. The success of stereo matching along two lines will be our indicator whether these two lines are corresponding epipolar lines.

Many different stereo matching methods exist (see Scharstein and Szeliski [5] for a survey). The stereo matching methods can be roughly divided to global and local methods. Since we are not interested in estimating an accurate per-pixel disparity, but only in line-to-line matching, we used dynamic programming stereo methods. Dynamic programming is the simplest and fastest global stereo algorithm, is relatively robust, and it gives the optimal solution for scanline matching.

3 Fundamental Matrix From Two Points

Given a pair of images of the same scene captured from two different view-points, together with two pairs of corresponding points, we wish to compute the fundamental matrix between the two images.

3.1 Problem Formulation

Given two images I_1, I_2 with two pairs of corresponding points (p_1, p_2) and (q_1, q_2), we want to estimate the fundamental matrix F between these images.

We start by seeking epipolar lines for each pair of corresponding points, using the consistency of the intensities along the lines. This consistency is computed from an optimal stereo matching along these lines. Each corresponding pair of points gives us an epipolar line in each images, and two pairs of corresponding points give us two epipolar lines in each image. Once we find the epipolar lines for the two pairs of corresponding points, the intersections in each image of the two epipolar lines gives the epipole. The third epipolar line needed for computing F is found from lines incident to the recovered epipoles.

In the next subsection we introduce the cost function for stereo matching along lines. In later subsections we introduce an iterative approach to compute the fundamental matrix using a RANSAC based algorithm for finding epipolar lines and epipoles.

3.2 Similarity Between Lines Using Stereo Matching

Given two images I_1, I_2, every point in each image resides on an epipolar line. The epipolar lines through a pair of corresponding points $p_1 \in I_1$ and $p_2 \in I_2$ are also corresponding epipolar lines. Given $p_1 \in I_1$ and $p_2 \in I_2$ our goal is to find the epipolar lines $l_1 \in I_1$ and $l_2 \in I_2$ through p_1 and p_2. We assume that the intensities along corresponding epipolar lines can be related through stereo disparities. This assumption is traditionally used in stereo, where depth

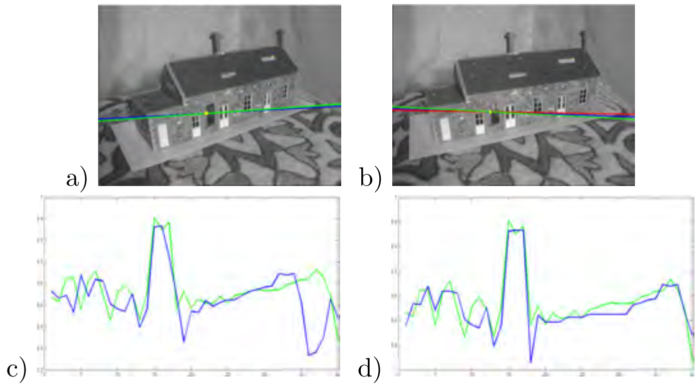


Figure 1: Matching epipolar lines using stereo matching. (a-b) Matching epipolar lines across two view. Computed epipolar lines by our method (green) are very close to the ground truth (blue). (c) The intensity profile of the two corresponding lines found by our approach. (d) Intensity profile after warping due to stereo correspondence.

is computed by matching points along corresponding epipolar lines. The similarity between two lines, l_1, l_2, is therefore defined as their optimal intensity correspondences (stereo matching) with an additional smoothness constraint.

Let x_i be the 2D coordinates of equidistant points along line l_1, and let y_i be the 2D coordinates of equidistant points along line l_2. The similarity between the two lines is based on their intensity similarities. It is formulated by the well known stereo matching equation [5, 6, 7] given the two lines l_1 and l_2, and the disparity d_i for every point x_i on l_1:

$$C(d; l_1, l_2) = \sum_{i=1}^{n} \phi(d_i; r) + \sum_{i=2}^{n} \psi(d_i; \alpha, \lambda), \tag{1}$$

where $\phi(d_i; r)$ is the data term, given as the truncated L_2

$$\phi(d_i; r) = min\{(I_1(x_i) - I_2(y_{i+d_i}))^2, r\}$$

and $r = 50^2$ [5]. The smoothness term ψ is over the disparities d_i given by:

$$\psi(d_i; \alpha, \lambda) = min(\lambda \cdot (d_i - d_{i-1})^2, \alpha).$$

where in our implementation we selected $\lambda = 2$ and $\alpha = 3$. The distance between two lines is the optimal disparity, d^*:

$$d^* = \arg\min_{d \in \mathbb{Z}^n} \left\{ C(d; l_1, l_2) \right\}, \tag{2}$$

Since we find the optimal disparities d_i^* in Eq. 2 using dynamic programming, the order constraint commonly used in stereo matching, $d_{i+1} \geq d_i$, is naturally obtained.

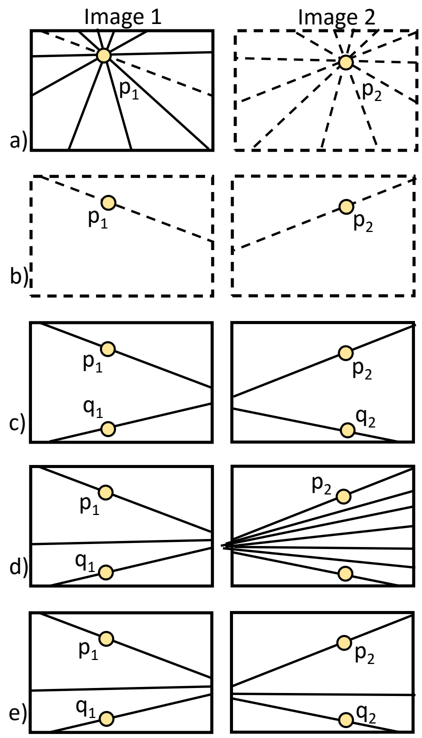


Figure 2: The Two-Points algorithm. (a) Given a pair of corresponding points, p_1 in Image 1 and p_2 in Image 2, we examine a pencil of lines incident to each point. (b) Each line from the pencil p_1, together with the line from the pencil of p_2, having the best stereo matching score, is selected as a possible corresponding pair of epipolar lines. (c) A pair of corresponding epipolar lines, passing through the second pairs of corresponding points, q_1 and q_2, is selected in the same manner. Their intersection in each image gives the third point - the epipole (which may be outside the image). (d) A bisector epipolar line is selected in Image 1, and a corresponding epipolar lines is searched for in the pencil of lines incident to the epipole in Image 2. (e) The last stage of the process gives us three corresponding epipolar lines, from which the epipolar line homography is computed.

3.3 Computation of Epipolar Line Homography

In this section we describe the steps taken for the computation of the epipolar line homography between two images, when two corresponding pairs of points in two images, (p_1, p_2) and (q_1, q_2), are given. The process is outlined in Fig. 2.

1. Through each pair of the selected points we generate a set of pairs of epipolar line candidates $\{l^i_{p_1}, l^i_{p_2}\}$. $l^i_{p_1}$ and $l^i_{p_2}$ will be considered candidates only if the second ($l^i_{p_2}$) is closest to the first ($l^i_{p_1}$) and the first is closest to

the second (mutual best matches), using the intensity similarity of Eq. 1 with the optimal disparities if Eq. 2. Following this step we have two sets of pairs of lines, one set for (p_1, p_2) and another set for (q_1, q_2). See Fig. 2.a.

2. Following steps are iterated:

   (a) A candidate pair of epipolar lines is sampled from each set generated in Step 2, see Fig. 2.b. Their intersections in each image, $e_1 = l^i_{p_1} \times l^j_{q_1}$ in Image 1, and $e_2 = l^i_{p_2} \times l^j_{q_2}$ in Image 2, are the hypothesized epipoles, see Fig. 2.c.

   (b) A third corresponding pair of epipolar lines, $\{l_{e_1}, l_{e_2}\}$, is found. The line incident to the epipole e_1 in image I_1 is taken as the bisector of the two lines that generated the epipole. The corresponding line, incident to the epipole e_2 in image I_2, is found by searching the closest line, in terms of stereo distance, from the pencil of lines incident to the epipole in Image 2. This is shown in Fig. 2.d. The epipolar line homography H from Image 1 to Image 2 is computed from the three pairs of corresponding epipolar lines, $\{l^i_{p_1}, l^i_{p_2}\}$, $\{l^j_{q_1}, l^j_{q_2}\}$, and $\{l_{e_1}, l_{e_2}\}$.

   (c) Another corresponding pair of epipolar lines is found, but this time a bisector epipolar line is selected in Image 2, and a corresponding epipolar lines is searched for in the pencil of lines incident to the epipole in Image 1. The epipolar line homography G from Image 2 to Image 1 is now computed. See Fig. 3.

   (d) The above epipolar line homographies will be correct only if $H = G^{-1}$, and each epipolar line in Image 1 should satisfy $\{l^i \approx GHl^i\}$. This can be checked by computing the area between l_i and GHl_i as in Fig. 4. To validate G and H we sum the above area for all lines.

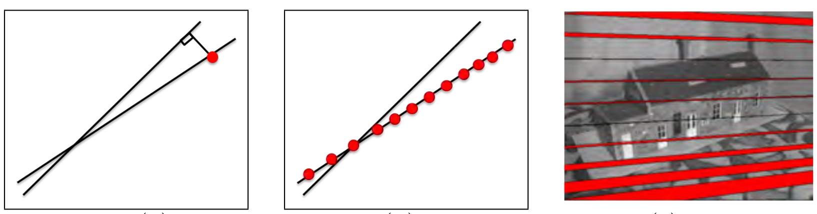
3. From all homographies computed in the previous step, we further examine only the 5% having the highest validation score. For these highly-ranked homographies we perform a full validation process.

4. Final Validation: Given the best homographies from the previous stage, we want select the the one that maximizes the quality of the stereo correspondences for many pairs of epipolar lines across the image. We sample a large number of epipolar lines through the epipole in one image and transfer them to the other image using the epipolar line homography. For each pair of epipolar lines we measure the stereo correspondence score according to Eq. 1 and sum over all lines.

5. Given the best homography H from the previous step, we compute the fundamental matrix: $F = [e_2]_\times H^{-1}$.

(a) (b) (c)

Figure 3: Initial validation of the epipolar line homogarphy. (a) The first image. The blue lines are the generating lines of the epipole and the red line is the third epipolar line, the angle bisector between them. The best correspondence to the red line is found in Image 2, and from these three lines the homography H from Image 1 to Image 2 is computed. (b) The second image. The red line is the bisector, for which a best correspondence is found in Image 1. This gives an independently estimated homography G from Image 2 to Image 1. (c) The composite homography GH should map epipolar lines to themselves. The red lines in the first image are transferred forward and backward using the composite homography. In an ideal case these lines should overlap.



(a) ‘ (b) (c)

Figure 4: The epipolar area measure. (a) The epipolar distance of one point. (b) Considering all the point on the line and evaluating their symmetric epipolar distance is the area between the two lines. (c) The total epipolar area is evaluated for several sampled epipolar lines.

3.4 Remarks

- To increase robustness by increasing the number of candidates, instead of examining only pairs of lines that are mutual best matches, we only require them to be in the top 2 matches of each other.
- If 3 (instead of 2) corresponding point matches are given, the algorithm can be significantly accelerated.
- The stereo matching is computed with an $O(N)$ time dynamic program.
- In the case where the original 2 point pair correspondences are not certain, this process can be embedded in an auxiliary RANSAC process sampling pairs of correspondences.

3.4.1 Complexity Analysis

Let the image sizes be $N \times N$. Through each point we sample $O(N)$ lines. We compare, using stereo matching, $O(N)$, each sampled line with the $O(N)$ lines sampled at its corresponding point. After $O(N^2)$ comparisons we are left with $O(N)$ candidate pairs of epipolar lines. As we are given two points in each image, each points generates $O(N)$ candidate epipolar lines, we get $O(N^2)$ possible intersections of epipolar lines, each such intersection is an hypotheses for the epipoles, e_1 and e_2.

For the third pair of corresponding epipolar line we select lines through the epipoles. In I_1 we select the bisector of the two epipolar lines that generated e_1. We find the best match for this line in I_2 by comparing it to $O(N)$ lines through e_2. This step can be skipped if there is a third pair of corresponding points r_1 and r_2, then $r_1 \times e_1$ and $r_2 \times e_2$ is the third epipolar line pair.

Validation is carried out by taking the epipolar area measure (See Fig. 4) of a fixed number of epipolar line in I_1 through e_1, $\{l^i_{e1}, GHl^i_{e1}\}$ with a complexity of $O(1)$.

For two points finding the possible pairs of epipoles together with finding the third line pair and validation takes $O(N^4)$ steps. When the algorithm is based on 3 pairs of corresponding points it requires at most $O(N^3)$ steps.

In practice, after filtering lines with little texture, a much smaller number of iterations is required.

4 Experiments

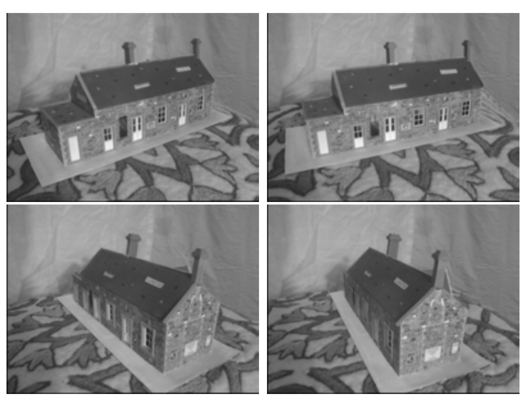


Figure 5: The house dataset from VGG is used for real data experiments. The dataset includes 10 images with various angles. Ground truth points and camera matrices are available.

We evaluated our method using stereo images of the house dataset by VGG, Oxford university [8]. The house dataset includes 10 images, representing different angles. The images are presented in Figure 5. We used every consecutive pair of images as a stereo pair, which results in 9 pairs. The size of the images is 768×576.

The quality of the computed fundamental matrices was evaluated using the symmetric epipolar distance [4] with respect to the given ground truth points. The baseline method is the 7-points algorithm. The 7-points method returns 3 possible solutions. In all experiments we selected for comparison the solution with the lowest symmetric epipolar distance. We also compared with the 8-points algorithms, which returned high-quality solutions after data normalization. Both the 7-points and 8-points algorithm were computed using the VGG toolbox[9].

For each pair of images we repeatedly executed 10 iterations. In each iteration we randomly sampled two pairs of corresponding points as input to our approach, seven pairs of corresponding points to use as input to the 7-point method and eight pairs of corresponding points to use as input to the 8-point method. The points were sampled so that they were at least 30 pixels apart to ensure stability. We computed the fundamental matrix using our method, the 7-point method, and the 8-points method. The symmetric epipolar distance was computed for each method. The points were sampled so that they were at least 30 pixels apart to ensure stability.

Table 1 shows the resulting fundamental matrix for each pair of images. Our method significantly outperforms the the 7-point algorithm. In 66% of the cases the symmetric epipolar distance is less than 3 pixels. The median error in our approach is 2.54. For the 8-points algorithm the median is 2.77 while the median in the 7-point algorithm is 25.8. Our approach depends on *global* intensity matching rather than on the exact matching of the points. Pairs $7, 8$ introduce a challenging stereo matching and as a result the quality of our method is effected. The global intensities in pairs 1-6 can be accurately matched and as a result the quality of the estimated fundamental matrix is high. Fig. 6 shows an example of rectification using our estimated fundamental matrix for pair number 1. Each horizontal line in the images is a pair of epipolar lines. Corresponding feature points are placed on the same epipolar lines.

5 Conclusions

We presented a method for estimating the fundamental matrix based on only two point correspondences, compared to traditional methods that need a 7 or 8 point correspondences. Two points are sufficient since we don't use only the point locations, but use them to search for corresponding epipolar lines. The correspondence of epipolar lines is determined by stereo matching using intensities along the entire lines. This is a powerful correspondence measure, as it is a global measure, and not a local measure as used for finding point correspondences.

Our 2-point method is very robust: Traditional approaches find a fundamental matrix that has a perfect match to the given corresponding points. Instead,

Image Pairs			7-Points	8-Points	2-Points
1			27.11	3.27	2.54
2			25.80	2.11	2.91
3			27.14	2.02	2.01
4			27.11	2.5	2.04
5			25.47	2.00	2.34
6			14.67	2.77	4.12
7			18.87	3.7	5.76
8			18.97	4.22	5.66
9			26.13	5.89	2.30

Table 1: The symmetric epipolar distance of the estimated fundamental matrix using the 7-points/8-points algorithm, and our 2-points algorithm. The distance is with respect to ground truth points. Accuracy of our 2-points algorithm is substantially higher than the 7-points algorithm and slightly better than the 8-points algorithm. The median error of our algorithm is 2.54. For the 8-points algorithm the median is 2.77 while for the 7-points it is 25.8.

our goal is to find a fundamental matrix that *globally* improves the stereo matching of the entire images.

In stereo reconstruction, intensity matching along epipolar line implies the matching of 3D points. In our approach we use the opposite direction. We assume that good correspondence of 3D points will give good stereo correspondences along epipolar lines. As far as we know, this is the first time that such an assumption has been demonstrated to be valid.

The need to for only two correct point correspondences makes an efficient RANSAC process. For example, if 50% of correspondences are outliers, and we require a propability of 99% that one subset will have no outliers, we will need to select 1,177 subsets of 8-points, 588 subsets of 7-points, and only 17 subsets of 2-points.

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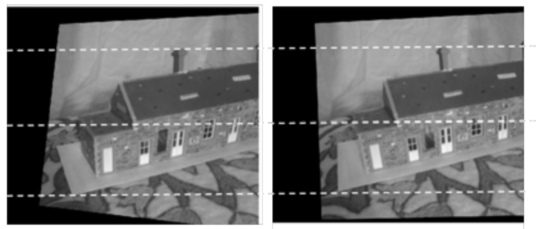

Figure 6: Rectification example using our 2-points method. The images are rectified after estimation of the fundamental matrix using our approach from only two pairs of corresponding points. Each horizontal line in the images is a pair of epipolar lines. Corresponding feature points are on the same epipolar lines.

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