QUANTUM COMPUTATION - EXERCISE 6

Due: 13 February 2006

1. Shor's Code

Give an exact analysis of what happens during an error of the type of a measurement (performed by the environment) in the $\{|+\rangle, |-\rangle\}$ basis of the leftmost bit in Shor's 9-qubit code. What are the possible states after the error occurs, and what happens during the error correction procedure.

2. Linear Code And Its Dual

Prove that the Fourier transform of the uniform superposition over the words in some linear code C in Z_2^n is the uniform superposition over the dual code $C^{\perp}$.

3. The No Quantum Majority Theorem

Let us consider the following hypothetical transformation. The input is a state of three qubits of the form $|\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle$. In case two or three of the states are equal (say, to alpha), the transformation outputs a first qubit in the state $|\alpha\rangle$, tensor with some (never mind which) two qubit state. In other words, the first qubit will contain the majority of the three states. Prove that such a majority transformation does not exist according to the laws of quantum mechanics.

4. A New Quantum Code

Some student in the class came to me and told me he found a quantum code that takes one qubit into 7 qubits. It encodes $|0\rangle$ into the state $|S_0\rangle$ and $|1\rangle$ into the state $|S_1\rangle$. The student claimed that the code has the following properties: First, the code can correct any error on a single qubit. Second, the reduced density matrix of $|S_0\rangle\langle S_0|$ to the first qubit, is strictly different from the reduced density matrix of $|S_1\rangle\langle S_1|$ to the first qubit. Can the student be correct? Prove rigorously!